

## What's So Bad About Infinite Regress?

Tony Roy (4/16/08, draft!)

Presumably I came from my parents, who came from theirs, etc. Many philosophers have held that series of this sort, where one thing depends on the next, etc. *cannot* continue to infinity. Or, at least, they have held that *vicious* infinite regress is impossible. But it's hardly clear what makes a regress vicious or benign, and so what justifies such evaluations. For certain ancients, it may have seemed sufficient justification, that the very idea of infinity is absurd. But in this day and age, with infinities routinely manipulated in mathematics, it is more difficult to reject infinity as such. In this paper, I assume the possibility of infinite series, and advance an account of their division into regresses, vicious and benign. On this account, a vicious infinite regress involves straightforward contradiction. Thus a theory which leads to vicious infinite regress is reduced to absurdity in the usual way.

Regress arguments have been part of philosophy since the time of Plato, and Plato's third man represents a great moment in the history of philosophy (*Parmenides* 132a-b; much discussion surrounds Vlastos 1954). Plato offers a theory that explains, in part, how distinct things can share a property or feature *F*. Against this theory, Parmenides raises his regress objection. It is not my aim to engage in questions of exegesis. Perhaps, though, Plato suggests a view on which (i) if some things are *F*, they are *F* only by participation in a form distinct from them, and (ii) if some things are *F* by participation in a form, the form is itself an *F* thing. On this view, there are forms, and things have their features by resembling them. Now suppose there are some *F* things. By (i), the *F* things are *F* by participation in a form distinct from them. By (ii), this form is an *F* thing. So the original things, and the form, are all *F* things. So by (i), they are all *F* by participation in a form distinct from them. By (ii), the new form is an *F* thing. So the original things, and the two forms, are all *F* things. Etc. It follows that there are infinitely many *F* forms. In Parmenides's

hands, the argument takes the form of a reductio: Plato's theory has the consequence that there are infinitely many *F* forms; this consequence is bad; so Plato's theory is to be rejected.

But this reasoning raises the question, what's so bad about infinite regress? why reject Plato's theory on the basis of this consequence? Perhaps certain ancients found the very idea of infinitely many *F* things absurd. Supposing that there are some *F* things, (i) and (ii) imply the denial of an additional premise, (iii) there are at most finitely many *F* forms. This may have been trouble enough for Plato. However, it is not so easy for us to reject infinity as such. Thus, e.g., contemporary Cantorian mathematics accommodates infinite quantities as a matter of course.<sup>1</sup> Even so, many have thought that there remains something wrong about (i) and (ii). The consequence is not merely that there are infinitely many *F* things, but that there is a *vicious infinite regress*. Infinite series are divided into those that are *regresses* and those (if any) that are not, and regresses are divided into those that are *benign* and those that are not. A regress that is not benign is *vicious*, and a vicious infinite regress is to be rejected. Like a donkey chasing a carrot suspended before its nose, in a vicious infinite regress, every step toward a goal somehow leaves the goal removed by another step.

But it is hardly clear what to make of this "eternal seeking," and even the division of infinite series into regresses, vicious and benign, is not well-understood. Given a regress, evaluations are typically treated as obvious or on-the-surface. Thus, e.g., Russell claims a regress I describe on p. 5 below (the *fundamental relation* regress), is "plainly vicious" (1911-12, 9). But other authors hedge their bets. Armstrong says versions of the regress are "either vicious or at least viciously uneconomical" (1989, 108), and J. Peterson that "while it may not be vicious, this regress... is unbelievable" (1991, 154). Insofar as it has application against a range of competing theories, Lewis treats the regress as a theoretical weakness, to be accommodated by one theory as much as by another (1983, 353-4). This seems odd if, as Russell thinks, the objection is fatal — for there is not much point contesting the viability of theories known to be dead. But these judgments are all made apart from an explicit account of what viciousness amounts to. As Alex Oliver suggests, "much of the trouble hinges on unclarities about the role of infinite regresses in

metaphysics, when they are vicious and when virtuous” (1996, 32). The problem is not merely when regresses are vicious, but also what viciousness amounts to. We need to know what makes a regress vicious, and why a vicious regress is to be rejected.

In this paper, I assume the possibility of infinite series as such, and advance an account of ones that are regresses, both vicious and benign. On this account, a vicious infinite regress involves straightforward contradiction. Thus a theory which leads to vicious infinite regress is reduced to absurdity in the usual way. In the first section, I sketch some familiar regress arguments, along with some initial attempts to understand them. In section two, I develop the account of infinite regress arguments. The third section applies this account back to simple versions of the regress arguments from section one. It is not my aim to dispute traditional evaluations of regress arguments. By and large, I think such evaluations are correct. Rather, my aim is to expose the way the regresses work. Given progress in other parts of logic, it is remarkable that regress arguments remain so poorly understood.

### I. Arguments and Evaluations

I begin this section by sketching some familiar regress arguments, and then turn to proposals for their evaluation. I do not pretend to decide larger issues about truth, human origins, God, and properties, or to give complete discussions of the proposals for evaluation. The larger questions depend on much more than the evaluation of these particular arguments; and I develop only the main outlines of proposals for evaluation. My aim is rather to put some issues on the table, to indicate something of their breadth and significance, and so to set up the discussion that follows.

Let us begin with a simple truth series. Say a declarative sentence in corner quotes names the proposition expressed by the sentence, and suppose we accept all expressions of the form,

(Tp) ‘ $p$ ’ is true iff  $p$

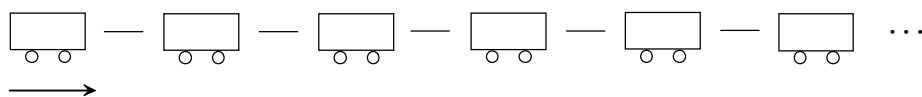
where the same declarative sentence is substituted for both instances of ‘ $p$ ’. Snow is white; so by an instance of (Tp), ‘snow is white’ is true; so by another instance of (Tp), ‘‘snow is white’ is true’ is true; etc. This may seem to be a paradigmatic example of a non-vicious series (see, e.g. Armstrong 1989, 54, cf. Carruthers 1982, 19). However, the situation is not always so simple.

Consider a hereditary series. Suppose no person is human unless the biological offspring of humans — that a person’s humanity depends on the humanity of his or her parents. Then if I am human, I have human parents; if they are human, they have human parents; etc. One might object, from a creationist perspective, that some human might have no parents at all, and thus no human parents. Or one might object, from an evolutionary perspective, that a sorites problem results from vagueness in the predicate ‘human’: perhaps a parent is the same kind as its child only up to some “tolerance” — so that members of different kinds might emerge gradually within a series. However, none of this matters. The question is rather, whether it is possible that the humanity of each child depend on the humanity of its parent. So I simply assume the condition. Romane Clark contends that this regress is vicious: evidence from biology to the side, a person’s humanity cannot depend on the humanity of her ancestors this way (1988, 377). Aquinas denies that this regress, or one very much like it, is vicious (*Summa Theologiae* I.46.2ad7; cf. Sanford 1984, 113-115).

Of course, for Aquinas, the situation changes when it is the motion of a rock, which depends on the motion of a stick, which depends on the motion of a hand, etc.

Whatever is moved must be moved by another. If that by which is moved be itself moved, then this also must needs be moved by another, and that by another again. But this cannot go on to infinity, because then there would be no first mover, and, consequently, no other mover, seeing that subsequent movers move only inasmuch as they are moved by the first mover; as the staff moves only because it is moved by the hand.

Since there cannot be an infinity of movers, Aquinas concludes that some mover is unmoved (and, of course, such reasoning is not without precedent in Aristotle).<sup>2</sup> I shall suppose the rock, stick, etc. are something like a series of train cars, each of which is accelerated by the one in front. Supposing a leftmost car accelerates, there is a series,



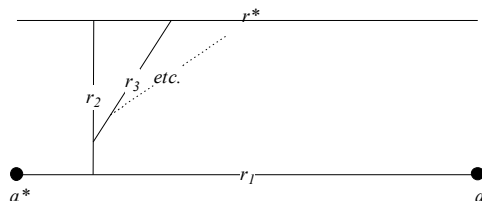
where each car is accelerated. Since the leftmost car accelerates, it is accelerated by the car in front of it; since the second car accelerates, it is accelerated by the one in front of it; etc. So the leftmost car, and the entire train, accelerates, but without an engine.

Finally, consider the “fundamental relation” regress, familiar from Russell’s *The Problems of Philosophy*. The third man regress is blocked by denying either that the form of  $F$  things is itself  $F$ , or that the form of  $F$  things is distinct from them. While, depending on cases, one or the other of these claims may be plausible against Plato, it is less easy to block a related difficulty involving the relation between forms and things. The problem has application against multiple solutions, both realist and nominalist, to the problem of universals. Russell’s version is directed at *resemblance* nominalism.

If we wish to avoid the universals *whiteness* and *triangularity*, we shall choose some particular patch of white or some particular triangle, and say that anything is white or a triangle if it has the right sort of resemblance to our chosen particular. But then the resemblance required will have to be a universal. Since there are many white things, the resemblance must hold between many pairs of particular white things; and this is the characteristic of a universal (1959, 96).<sup>3</sup>

On the resemblance theory, there are no universals, and a thing is  $F$  only if it appropriately *resembles* a standard exemplar. Russell claims that resemblance must therefore be universal. However, as he suggests in another place, we might apply the same analysis again, and say a relation is a *resemblance* just in case it resembles a standard exemplar of resemblance. “It is obvious, however,” he says, “that such a process leads to an endless regress... and such a regress is plainly vicious” (1911-12, 9 — note that his 1959, quoted above, was first published around the same time, in 1912). Consider some color patch  $a$ ;  $a$  is white only if it appropriately resembles a standard white patch  $a^*$ . So consider some relation  $r_1$  between  $a$  and  $a^*$ ; on the current theory,  $r_1$  is a resemblance only if it appropriately resembles a standard resemblance  $r_1^*$ . So consider some relation  $r_2$  between  $r_1$  and  $r_1^*$ ; presumably,  $r_2$  is a resemblance only if it appropriately resembles a standard resemblance  $r_2^*$ . So consider some relation  $r_3$  between  $r_2$  and  $r_2^*$ ; etc. Maybe there is just

one standard resemblance, so that  $r_1^*$ ,  $r_2^*$ , etc. are the same. Still,  $r_1$  is a resemblance only if  $r_2$  is a resemblance;  $r_2$  is a resemblance only if  $r_3$  is a resemblance; etc. In this case, the situation might be pictured as follows:



Each relation requires another for it to be a *resemblance*. Thus this resemblance nominalism has the consequence that there is an infinite series of resemblances. Russell, at least, thinks it is a vicious regress, and that resemblance nominalism is therefore to be rejected. Similarly if, on a realist view, things *participate* in forms, one might think that the relation between a form and thing should itself participate in participation; etc. As I say above, not all agree that these series are vicious.

In a recent paper, Oliver Black characterizes an infinite regress argument as a *reductio* with four premises: three from which it follows that there is an infinite series of a certain sort, and a fourth according to which there is no such thing (1996; cf. Sanford 1975, 520).<sup>4</sup> For some property  $F$ , begin by supposing *existence* — that there is some  $F$  thing.

$$(E) \quad (\exists x)Fx$$

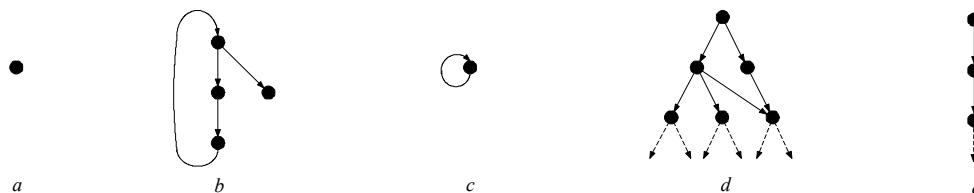
In the relatively simple case of the hereditary series, the thing is a human. But also, for some relation  $R$ , suppose *generation* — that any  $F$  thing stands in relation  $R$  to an  $F$  thing.

$$(G) \quad (\forall x)[Fx \supset (\exists y)(Fy \ \& \ Rxy)]$$

In the case of the hereditary series, each human is the biological *offspring* of others. For the third premise, where some dots  $\bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet$  are related by  $R$  when connected by an individual arrow, say they are related by  $*R$  when there is a path along the arrows from one to the other. In the case of the hereditary series,  $Rxy$  just in case  $y$  is a *parent* of  $x$ , and  $*Rxy$  just in case  $y$  is an *ancestor* of  $x$ .  $*R$  is thus the *ancestral* of  $R$ .<sup>5</sup> Given this, we require that nothing bears  $*R$  to itself — that  $*R$  is irreflexive, and so a (strict) *partial order*.

$$(P) \quad \sim(\exists x)*Rxx$$

To see how these conditions work, consider (a) - (e) below.



Where dots are  $F$  things, and related by  $R$  when they are connected by an individual arrow, they are related by  $*R$  when there is a path along the arrows from one to the other. By (E), there is at least one dot; (a) - (e) each satisfy (E). (G) requires that each dot bear  $R$  to some dot; (a) fails this condition because the only dot does not bear  $R$  to any dot; (b) fails because the last dot on the right-hand branch bears  $R$  to none; (c) meets this condition insofar as the only dot bears  $R$  to a dot — namely itself. (P) rules out cases where (E) and (G) are satisfied in “ $R$ -circles”; the only dot in (c) bears  $R$  to itself; so it bears  $*R$  to itself; so it violates (P); the dots in the main branch of (b) all violate (P) insofar as there is a path along the arrows from any dot to itself. Thus (E), (G) and (P) require that series continue “downward” as in, e.g., (d) or (e).

One might think we have just shown that (E), (G) and (P) guarantee the existence of infinite descending paths, but this is not quite right. To see this — and that (E), (G) and (P) do in fact guarantee the existence of such paths, say a path or *sequence*  $S = \langle s_1, s_2, \dots \rangle$  is a function whose domain is an initial segment of the positive integers (an ordinal), with  $f(1) = s_1$ ,  $f(2) = s_2$ , etc. Where  $n'$  is the successor of  $n$ , say a sequence  $S$  is an  $R$ -series just in case any  $s_n, s_{n'} \in S$  are such that  $R s_n s_{n'}$ ; and say an  $R$ -series is an  $RF$ -series just in case each of its members is  $F$ . By (E), there is an  $a$  such that  $Fa$ ; so  $\langle a \rangle$  is an  $RF$ -series. By (G), there is a  $b$  such that  $Fb$  and  $Rab$ , so  $\langle a, b \rangle$  is an  $RF$ -series. By (P),  $a$  and  $b$  are distinct. Etc. So there is a one-member series, a two-member series — and there are infinitely many such series. But just as it is one thing to say there are infinitely many integers, and another that some integer is infinite, so it is one thing to say there are infinitely many such series, and another that some series is infinite; so to show that there are infinitely many series (always continuing downward) is not itself to show that any series continues infinitely. But suppose we are given *linearity* — that distinct objects on paths from  $a$  are always connected by  $*R$  in one direction or another.

$$(L) \quad (\forall x)(\forall y)[(*Rax \ \& \ *Ray \ \& \ x \neq y) \supset (*Rxy \vee *Ryx)]$$

Elements occurring on distinct branches are not connected by  $*R$ ; (L) thus requires that there be no distinct branches, so that the picture is as in (e) above. In this case, a “union” of the infinitely many finite paths from  $a$  is a path of infinite length. Similarly, even without (L), it is possible to “choose” a path of this sort. Supposing a restriction to some subclass of paths whose members do not branch, again, a “union” of infinitely many finite paths from  $a$  is a path of infinite length.<sup>6</sup> With or without (L), then, (E), (G) and (P) in fact require the existence of an  $RF$ -series with infinitely many distinct members.

But (E), (G) and (P) are not therefore problematic. Or, at least, (E), (G) and (P) do not by themselves lead to contradiction. To demonstrate their consistency, it is enough to find an interpretation on which they are all true. But (E), (G) and (P) are, e.g., true on the integers, along with (L), when  $R$  is *successor* and  $F$  is *integer*. So they are consistent. Black simply adds *finitude*,

$$(F) \quad \text{There is no infinite series of } Fs \text{ all related by } R$$

as a fourth premise for reductio. Then a regress is vicious just when (E), (G) and (P) imply that there is an infinite series of  $F$ s related by  $R$ , and (F) that there is no such thing.<sup>7</sup> Given this premise, there is no problem about reaching a contradiction, but there is a problem about the basis for the premise. (F) tells us that there is no regress, but leaves us wondering why. Maybe something like (F) is true in every case where there is a vicious infinite regress. Even so, (F) is question-begging insofar as the question is what’s so bad about infinite regress — for we want to know why there is no such series. Thus Black does not answer the question we have asked.

For now, let me observe that the intuitive difficulty about vicious infinite regress arises out of our very reasons for thinking there is a regress in the first place. Somehow, the elements of a vicious infinite regress are introduced toward an end which remains forever unattained. The problem is precisely that some supposed end remains unattained. If this is right, (F) or something like it, is a consequence of reasons for regress, and so not independent of them. Insofar as (E), (G) and (P) are consistent — and supposing that what’s bad about infinite regress exhibits itself in

contradiction — (E), (G) and (P) are therefore not the whole story about reasons for vicious infinite regress.

Romane Clark suggests that a vicious infinite regress is characterized by “downward dependence” (1988). From the standard truth table, the material conditional,  $\mathcal{P} \supset \mathcal{Q}$ , is true iff  $\mathcal{P}$  is false or  $\mathcal{Q}$  is true; this condition does not require *dependence* between  $\mathcal{P}$  and  $\mathcal{Q}$ ; so any such conditional leaves it open whether  $\mathcal{Q}$  depends on  $\mathcal{P}$ ,  $\mathcal{P}$  depends on  $\mathcal{Q}$ , or neither. Thus (G) leaves it open whether one member of a series depends on another and, if there is dependence, what the direction of dependence might be. On Clark’s account, what is missing from Black’s view (though Clark writes before Black and so does not directly respond to him) is a dependence left out by the material conditional in (G). Clark says a relation  $R$  is (*upward*)  $F$ -*preserving* if, for any  $a$ ,  $Rab$  and  $Fb$  guarantee  $Fa$ . Then,

Something is *conditionally F* just in case there is something to which it stands in an  $F$ -preserving relation  $R$  which induces a partial order.... If this is the only way a thing comes to be  $F$ , if something is *only conditionally F*, then, with respect to  $F$ , it is *downward dependent* on its  $R$ -related heredity.... Something is *categorically F* just in case it is  $F$  but not only conditionally so (1988, 173).

If the only way a thing can be  $F$  is to have  $R$  to an  $F$  thing, it is only conditionally  $F$ . A thing is categorically  $F$  iff it is  $F$  but not only conditionally so. Given this, on Clark’s view, the typical infinite regress argument is developed as a reductio.  $R$  induces a partial order, so (P) remains as above. There is a premise according to which something is categorically  $F$ ,

(E\*)  $(\exists x)(x \text{ is categorically } F)$

or, equivalently,  $(\exists x)(x \text{ is } F \ \& \ x \text{ is not only conditionally } F)$ . Finally, for some upward  $F$ -preserving relation  $R$ , it is sufficient that a target thesis implies that whatever is  $F$  is only conditionally  $F$ . This condition may take the form,

(G\*)  $(\forall x)[Fx \text{ only if } (\exists y)(Fy \ \& \ Rxy)]$

If these premises collapse into (E), (G) and (P) then, as before, there is no contradiction. However, the conditional in (G\*) is not to be understood materially. (G\*) goes beyond (G)

insofar as it constrains the direction of dependence, requiring that each member of a series depends on the next. Clark leaves this notion at an intuitive level. Let us say we understand, and do so as well.

Even so, it is not clear how to take Clark's proposal. His initial idea seems to be that (E\*) is typically given and leads to contradiction with (G\*). As developed above, (E\*) does conflict with G\* — from (G\*), whatever is *F* is only conditionally *F*, and from (E\*) something is *F* but not only conditionally *F*. Unfortunately, it is hard to see how (E\*) is given in the ordinary case. It may be given that someone (really!) is human; but it is not given that her humanity does not depend on an infinite series. For this, we need to know what's so bad about infinite regress. But Clark also suggests (on the same page as the passages quoted above) that every *F* thing is categorically *F*. If this is right, (E\*) collapses into (E), and we get the contradiction for reductio insofar as (E) is itself inconsistent with (G\*) — insofar as something is supposed to be both *F* and only conditionally *F*.

But (G\*) does not contradict (E). For a relatively simple case, consider a series of cats stalking a mouse, where any cat in the series bites the tail in front of it just in case its tail is bitten from behind.



(In case of worry about cruelty to animals, substitute some mechanical device as below.) Now suppose a first cat bites the mouse's tail; then there is a second cat which bites the first cat's tail; so there is a third cat which bites the second cat's tail; etc. So there are infinitely many cats. Supposing, as we have, that infinity is not itself problematic, there should be no objection to an infinite series of cats as such. And there should be no objection to cats that bite just in case they are bitten. Insofar as a cat bites only if it is bitten, it is natural to think that biting is downward dependent. But there is no contradiction supposing that the first cat bites the mouse's tail. If the cats are arranged in a circle, either each bites, or none bites. Similarly, in an infinite series, either each bites or none bites. And the options seem equally plausible. God could create the series of

cats “all at once” in either state. All that is required for biting is that each cat be in proximity to one that bites — where, seemingly, God could create them all that way. And similarly for not biting. Given this, the downward dependence of  $F$  is not, in general, sufficient for the conclusion that not- $F$ . So (G\*) does not imply the negation of (E). I return to this case below.

For now, notice that downward dependence (supposing we understand it) does distinguish the truth series, snow is white, ‘snow is white’ is true, ‘‘snow is white’ is true’ is true, etc. from other, plausibly vicious, regresses. It is natural to say that this series is “upward,” rather than “downward” dependent — and we might therefore call it a “progress” or “progression” rather than a “regress” or “regression.” Perhaps downward dependence is necessary and sufficient for *regress*. And there may be a kind of series which does not involve dependence at all, and so is neither regress nor progress. But downward dependence does not distinguish, say, the motion of a rock which depends on the motion of a stick, etc., from my humanity, which depends on my parents’, etc. Each of these seems to involve downward dependence. As above, Clark uses his condition to conclude that the hereditary series is vicious. So if the traditional evaluations, on which not both are vicious, are correct, then downward dependence is not sufficient to distinguish regresses that are vicious from those that are benign. And again, if there is no problem about the regress of cats, Clark’s condition is not sufficient to distinguish benign from vicious regresses. Having distinguished regresses from other series by means of Clark’s condition, it remains to say which regresses are vicious, and which are benign.

The inadequacy of these proposals seems to cast us back on formulations, like one I use above, according to which the elements of a vicious infinite regress are introduced toward an end which remains forever unattained. Thus, e.g., Sanford, commenting on Passmore, and a remark by Geach according to which the real trouble with vicious regress “arises already at the first step,” considers that a theory which leads to regress makes some promise, and says,

The real trouble arising already at the first step is that of making no progress. We should see this straight away. If we do not, we may see it after realizing that no number of steps, not even an infinite number, makes any progress toward explaining, defining, analyzing, or

accounting for something. Drawing attention to an infinite regress can thus have a function even though the real trouble is not due to the regress (1984, 96; cf. Passmore 1961, 19-37, and Geach 1979, 100-101).

Related points are sometimes made in terms of human capabilities: a series must fail to deliver on some promise, because it is impossible for humans to complete an infinite series. No doubt, it *is* impossible for humans to complete at least certain infinite series of tasks.<sup>8</sup> But the cases we have considered, at least, do not have to do with human capabilities; they have rather to do with the existence of truths, ancestors, movers, and relations. And it is not necessary to cast the current proposal in terms of human capabilities: a regress is vicious when even an infinite series fails to deliver on some promise. Perhaps it is obscure how “lack of progress” is to be distinguished from “downward dependence” — if one element of a series depends on the next just because there is no progress, lack of progress may seem to go hand-in-hand with downward dependence. However, I think these suggestions are on the right track. It is the task of the next section to develop and defend this claim.

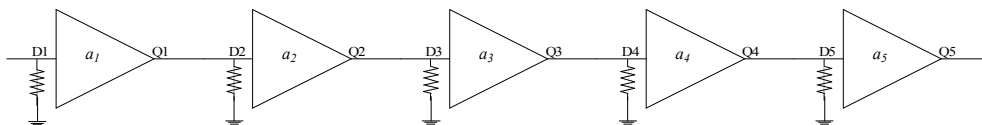
## II. A Positive Theory

In this section, I propose a theory, exhibit it in application to some relatively well-understood physical models, and comment on the result. Paradoxically, one advantage of the physical cases is that they can be relatively complex while, at some level, philosophical examples may be so simple as to obscure distinctions that matter. The physical examples have the advantage that devices are subject to simple laws, and make possible different series from a small set of primitives. To manipulate cases, we will need a little math and physics; however, I do my best to keep it at an accessible basic level. We return to philosophical examples in the following section.

Recall that an *R*-series is a sequence with adjacent members related by *R*. Developing Sanford’s suggestion that the real trouble with infinite regress “is that of making no progress,” I propose that a valid infinite regress argument arises when premises imply that there is an *R*-series which both is, and is not, adequate to some end. The reasoning involves considerations of three

sorts: (i) For some property  $F$  and relation  $R$  with irreflexive ancestral  $*R$ , there are *adequacy* premises according to which there is an  $R$ -series whose first member is  $F$ . So far, it may be open whether the series has just one member, or many. (ii) There are *underlying* premises which specify some features of the members. Ordinarily, the features are “relatively intrinsic” insofar as they do not depend on relations to other members of the series. And (iii) there are *linking* premises that fix some functional relation between the underlying features and adequacy. As we shall see, a lot hangs on the nature of this linking relation. An  $R$ -series is either finite or (countably) infinite. If it follows from (ii) and (iii), by induction or whatever, that no finite  $R$ -series is adequate, then the series is a regress and infinite. If, in addition, it follows from (ii) and (iii) that neither does an infinite series satisfy (i), then the original premises are inconsistent, the regress is vicious, and at least one premise must be rejected. As we will see, however, not every series characterized by such premises is a regress, and not every regress is vicious.

To put some flesh on these bones, let us begin with the series of cats turned around so that the mouse bites the first cat, who bites the second, etc. Imagine that the force of each cat’s bite is in proportion to the force with which it is bitten. Then the cats are like a series of electronic *amplifiers*. An amplifier is a device with *input voltage*  $D$ , *output voltage*  $Q$ , and *gain*  $a$ , where  $Q = a * D$ . Such devices may be connected in series as follows.



As configured above,  $D1$  is fixed at zero (ground), so  $Q1 - Q5$  are zero as well. If there is a power source so that  $D1$  has some non-zero value  $V$ , we may reason from values at one amplifier to values at the next. For any  $n$ ,  $Q_n = a_n * D_n$  and, since they are connected directly,  $Q_n = D_{n+1}$ . So,

$$\begin{aligned}
 Q5 &= a_5 * D5 \\
 &= a_5 * Q4 = a_5 * a_4 * D4 \\
 &= a_5 * a_4 * Q3 = a_5 * a_4 * a_3 * D3 \\
 &= a_5 * a_4 * a_3 * Q2 = a_5 * a_4 * a_3 * a_2 * D2 \\
 &\quad \vdots \\
 &= a_5 * a_4 * a_3 * a_2 * a_1 * D1
 \end{aligned}$$

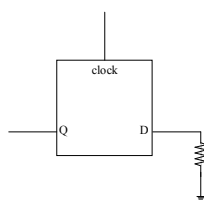
where equalities in the horizontal direction are because  $Q_n = a_n * D_n$ , and in the vertical because  $Q_n = D_n'$ . And, in general,  $D_1 * a_1 * a_2 * \dots * a_n = Q_n$ . Thus the state of each amplifier is fixed once values are given for  $D_1$  and the gains. And similarly for the cats, where inputs and outputs are like forces, and gains the proportions with which cats react to being bitten.

And similarly in the infinite case (if you like imagine that, through a miracle of miniaturization, amplifiers get progressively smaller, so that the series fits into a finite space). To make the case more specific, suppose the gains are arranged into a series  $\sqrt[2]{a}, \sqrt[4]{a}, \dots$  where for any  $n$ ,  $a_n = a^{1/2^n}$ . One reason for this choice is to keep the arithmetic relatively simple: For a series of  $n$  members, the sum  $1/2^1 + 1/2^2 + \dots + 1/2^n = 1 - 1/2^n$ ,<sup>9</sup> and a product,  $a^{1/2^1} * a^{1/2^2} * \dots * a^{1/2^n}$  is equal to  $a^S$ , where  $S$  is the sum of the exponents, so  $a^{1/2^1} * a^{1/2^2} * \dots * a^{1/2^n} = a^{1-1/2^n}$ . For an infinite series, the sum,  $1/2^1 + 1/2^2 + \dots$  is the limit of finite partial sums, so  $1/2^1 + 1/2^2 + \dots = 1$ , as  $1 - 1/2^n$  approaches 1; and a product  $a^{1/2^1} * a^{1/2^2} * \dots$  is the limit of finite partial products, so  $a^{1/2^1} * a^{1/2^2} * \dots = a^1 = a$ , as the sum of exponents approaches 1.

Now suppose some amplifier system is such that: (i) There is an infinite  $R$ -series of amplifiers with  $D_1 = V$ . (ii) Individuals in the series are such that for any  $n$  in the series's domain,  $a_n = a^{1/2^n}$ . And (iii) individuals in the series are linked so that, as above, with  $\Pi$  an extended *product* function, for any  $n$ ,  $Q_n = \Pi[a_1, a_2 \dots a_n] * D_1$ . Then, in the infinite case, where  $Q_\omega$  is not a physical value in the series, but rather the limit of the outputs,  $Q_\omega = \Pi[a_1, a_2, \dots] * D_1$ . From (ii) and (iii), with the arithmetic from above, for any  $n$ ,  $Q_n = a^{1-1/2^n} * D_1$ , and  $Q_\omega = a * D_1$ . So with (i),  $Q_n = a^{1-1/2^n} * V$  and  $Q_\omega = a * V$ . Given a value for  $V$ , then, values for  $Q_n$  and  $Q_\omega$  are fixed. And there is nothing inconsistent about this: (ii) and (iii) give us a functional connection between the value of  $D_1$  and the values of  $Q_n$  and  $Q_\omega$ ; but they do not thereby fix the value of  $D_1$ , and so do not force contradiction with (i). Rather, (ii) and (iii) make other values a function of  $D_1$ . This will be my standard example of an infinite progress. It is infinite because (i) with (ii) and (iii) fix values "all the way out." It is a progress insofar as values for later members are determined by ones before, not the other way around. (With the details of this case under our belts, details for others should be relatively straightforward.)

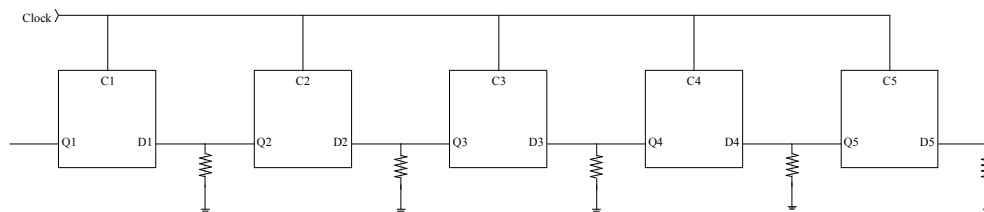
We get closer to the example of cats which simply bite or not by setting each gain equal to one, and treating the amplifiers as two-state systems. For this, it is enough to let a voltage less than or equal to some cutoff (say, ground) be the value *low* ( $L$ ), and any voltage above it *high* ( $H$ ) — where amplifiers correspond to cats, output  $H$  to biting, and  $L$  to not biting. Then everything works as before. With the gains equal to one, for any  $n$ ,  $D1 = Qn = Q\omega$ . So if  $D1 = H$ , all the values are  $H$ ; if  $D1 = L$  all the values are  $L$ . And there is no contradiction.

For a regress, let us turn the cats back around so that they stalk the mouse. Again, we will model the cats with amplifiers. As a first step, however, consider a bucket brigade in which members pass their bucket, if they have one, to the next on command of a captain (who is like the coxswain). This works like a series of electronic “flip-flops” arranged to form a “shift-register.” Such systems “shift” values so that we will be able conveniently to isolate the effects of one stage



upon the next. The input  $D$  and output  $Q$  of a simple *flip-flop* take either of the states high  $H$  or low  $L$ . The clock input cycles on and off. The flip-flop is such that the state of  $Q$  after  $c+1$  clock cycles is equal to the state of  $D$  after  $c$  clock cycles. As pictured to the side,  $D$  is fixed at  $L$  and, whatever its initial

condition,  $Q$  is therefore  $L$  at every clock cycle after the first. Such flip-flops may be arranged in series, to form a *shift-register*.

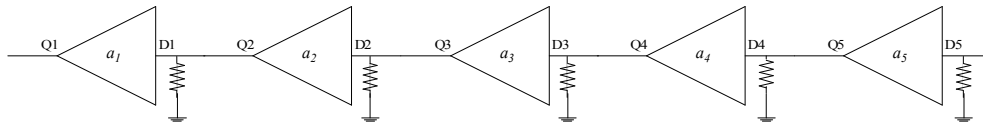


Supposing, e.g., that  $Q1 - Q5$  are initially in the states  $LHLLH$ , after one clock cycle they are  $HLLHL$ , after two  $LLHLL$ , after three  $LHLLL$ , after four  $HLLLL$  and after five,  $LLLLL$ . Thus the initial pattern is “shifted” through the structure from right to left, with the “vacated” locations taking the value from  $D5$ . If  $Q1$  is connected around to  $D5$ , the pattern shifts round and round, returning to its initial state every five clock cycles. Thus high values are like buckets passed from one member to the next. (In my youth, I constructed a display which used shift registers to cycle

lights around like this in different patterns.) Say a pattern is *stable* just in case all output states remain the same from one clock cycle to the next. Then *LLLLL* is the only stable pattern for the above finite device. If Q1 is connected to D5, then *LLLLL* and *HHHHH* are stable. And, if the series is infinite, all *H*, and all *L*, are stable.

This infinite case is related to “Hilbert’s Hotel” examples as applied by William Lane Craig against the possibility of an actual infinite. Thus, e.g., consider a hotel with infinitely many rooms having a guest and bar of soap in each room.<sup>10</sup> Say the occupant of the first room sells his bar of soap, receiving another from the occupant of the second room, who receives one from the occupant of the third, etc. Craig would find it mysterious that, no matter how many bars of soap the first occupant sells, and how much profit is thereby made, no room is left without a bar of soap. He concludes that Hilbert’s infinite hotel is not actually possible. But there is a straightforward response: An infinite series is “unbounded” in the sense that it has no last member, and so can be put in correspondence with the integers. There are ways of removing individuals from such a series, some of which preserve this property, and some of which do not. So long as unboundedness is preserved, as when just the initial bar (or every other bar) is removed, the series remains infinite — and so capable of correspondence with the integers (and rooms); if unboundedness is not preserved, as when, say, all the members after the tenth are removed, the series is no longer infinite. Thus the account of infinity *explains* why subtraction from an infinite series works the way it does — whether a series remains infinite after subtraction does not have so much to do with the “number” of members removed, as with the way they are removed and the structure of the remaining series. Craig is right to hold that the possibility of Hilbert’s Hotel entails the possibility of shifting soap. However, given our assumption that the hotel is infinite, there is no mystery about shifting the soap: we admit the consequence, but deny that it is absurd. Similarly, having assumed the possibility of infinite series as such — and so having already rejected Craig’s conclusion against infinity, we commit ourselves to the possibility of “shifting from infinity” as above.

Removing the clock gets us closer to the series of cats. For this, let us return to a series of amplifiers, now reversed to reflect the direction of the cats.



Though we no longer “see” the pattern being shifted through the structure, this device works very much like the shift register. For the above finite system, all  $L$  is the only stable state. If Q1 is connected around to D5, or the series is infinite, all  $H$  and all  $L$  are stable. Accounting for the gains,

$$\begin{aligned}
 Q1 &= a_1 * D1 \\
 &= a_1 * Q2 = a_1 * a_2 * D2 \\
 &= a_1 * a_2 * Q3 = a_1 * a_2 * a_3 * D3 \\
 &\vdots \\
 &= a_1 * a_2 * a_3 * a_4 * a_5 * D5
 \end{aligned}$$

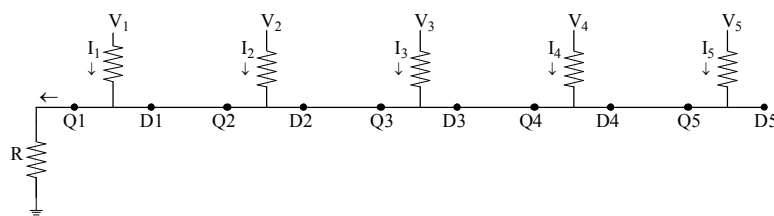
where equalities in the horizontal direction are by  $Qn = a_n * Dn$ , and in the vertical by  $Dn = Qn'$ . As configured above,  $D5 = 0$ , so  $Q1 - Q5$  are 0 as well.

Now suppose, (i) there is an  $R$ -series of amplifiers with  $Q1 = V$  for some  $V > 0$ . (ii) Individuals in the series are such that for any  $n$  in the series’s domain,  $a_n = a^{1/2^n}$ . And (iii), as above, individuals are linked so that for any  $n$ ,  $Q1 = \Pi[a_1, a_2 \dots a_n] * Dn$ ; in addition, if there is a *last* member of any such series, it has input 0. Then for an infinite series,  $Q1 = \Pi[a_1, a_2 \dots] * D\omega$ , where  $D\omega$  is the limit of the inputs. From (ii) and (iii), for any  $n$ ,  $Q1 = a^{1-1/2^n} * Dn$ , and for an infinite series,  $Q1 = a * D\omega$ . So by (i),  $V = a^{1-1/2^n} * Dn$ , and if the series is infinite,  $V = a * D\omega$ . But for a series with just  $n$  members,  $Dn = 0$  and, since  $V > 0$ , given  $V = a^{1-1/2^n} * Dn$ , this is impossible. We can thus reason from the initial value  $V$ , to a positive value for  $Dn$  — where this contradicts the assumption that the series has just  $n$  members, and so  $Dn = 0$ ; since the assumption that the series is finite leads to contradiction, the series is infinite. Or, put the other way around, in finite cases, we can reason from  $Dn = 0$  to a zero value for  $V$ , where this contradicts adequacy. Thus (ii) and (iii) suffice to fix a value for  $Q1$ , and there is room for contradiction. When the series is infinite, however, contradiction evaporates: we have  $V = a *$

$D\omega$ , but without a value given for  $D\omega$ . So again (ii) and (iii) do not determine a value for  $Q1$ , and there is no room for contradiction. This will be my standard example of a benign infinite regress. It is infinite because the value at every finite stage requires one at the next. It is a regress because, in finite cases at least, later stages suffice to determine values at ones before. It is benign since, in the infinite case, (ii) and (iii) are consistent with (i).

Again, we get closer to the original example of the cats by setting the gains equal to one, and treating the amplifiers as two-state systems. With the gains equal to one, for any  $n$ ,  $Q1 = Dn = D\omega$ . Suppose  $Q1 = H$ . Then, for an  $n$  member series,  $Dn = H$ ; but in a finite  $n$ -member series,  $Dn \neq H$ ; this is impossible, so the series is not finite. However, there is no problem in the infinite case, as nothing prevents  $D\omega$  from taking the same value as  $Q1$ . So each of the cats can bite, and the regress is benign.

For a vicious regress, begin with a series of tributaries flowing into a single river, where we are interested in total flow from the tributaries. This is like a summation of electronic currents.<sup>11</sup> Given some path to ground, in this case, an individual consists of a voltage source and resistor, where the voltage is regulated so that potential across the resistor is constant and current through the resistor is thus fixed at some constant value. Such individuals may be strung together as follows.



For any  $n$ , the total current at  $Qn$  is the sum of  $I_n$  and  $Dn$ . So in this case,

$$\begin{aligned}
 Q1 &= I_1 + D1 \\
 &= I_1 + Q2 = I_1 + I_2 + D2 \\
 &= I_1 + I_2 + Q3 = I_1 + I_2 + I_3 + D3 \\
 &\vdots \\
 &= I_1 + I_2 + I_3 + I_4 + I_5 + D5
 \end{aligned}$$

where equalities in the horizontal direction are by  $Qn = I_n + Dn$ , and in the vertical by  $Dn = Qn'$ .

As here configured,  $D5 = 0$ , and  $Q1$  is just the sum of  $I_1 - I_5$ .

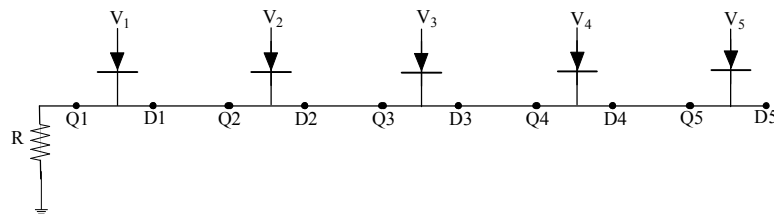
Now suppose, (i) there is an *R*-series of current sources with  $Q1 = I_0$ . (ii) Individuals in the series are such that for any  $n$ ,  $I_n = 1/2^n$ . And (iii) individuals are linked so that for any  $n$ ,  $Q1$  is the extended sum,  $\Sigma(I_1, I_2 \dots I_n) + Dn$ ; in addition, if any such series has a last member, its input  $Dn = 0$ . Similarly, in the infinite case, the *only* sources of current are from individual members of the series, so  $Q1 = \Sigma(I_1, I_2, \dots)$ . Now suppose  $I_0 > 1$ . Then, in any finite case,  $I_0 = 1/2^1 + 1/2^2 + \dots + 1/2^n = 1 - 1/2^n$ . But,  $1 - 1/2^n < 1$ , and this is impossible. So the series is infinite. But  $\Sigma(1/2^1, 1/2^2 \dots) = 1$ .<sup>12</sup> So again there is contradiction. This will be my standard example of a vicious infinite regress. In both finite and infinite cases, later stages suffice to determine values at ones before. It is vicious since in neither the finite nor infinite case are (ii) and (iii) consistent with (i).

So, on this view, a typical regress argument has adequacy, underlying, and linking premises. In a progress, underlying and linking premises do not fix relevant features for the first member. Rather, underlying and linking premises, with claims about the first member, fix features for ones that follow. In a regress, underlying and linking premises do fix relevant features for the first, at least in finite cases. Thus there is grist for contradiction with adequacy. A regress is vicious when underlying and linking premises are inconsistent with adequacy in both the finite and infinite cases.<sup>13</sup>

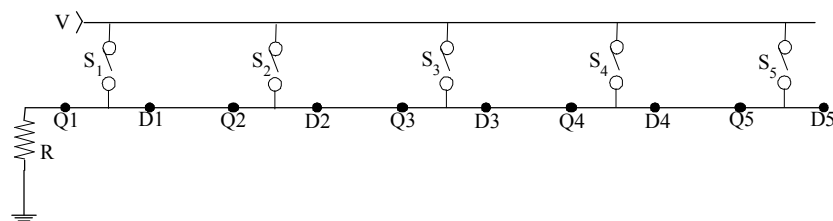
Notice that the essential difference between benign and vicious cases is not that benign regresses somehow involve infinite products, and vicious regresses summations. If, e.g., in a regress of amplifiers (or cats), voltages were *augmented* by some value  $a_n$ , then for any  $n$ ,  $Q1 = \Sigma(a_1, a_2 \dots a_n) + Dn$ , and in the infinite case,  $Q1 = \Sigma(a_1, a_2 \dots) + D\omega$ . With  $Dn = 0$  for a last member, there might be contradiction in every finite case, without contradiction when the series is infinite; so the series related by summation would be benign. Similarly, series related by product functions might be vicious.<sup>14</sup> The important point is rather about the way linking functions “collect” contributions of the members. In a vicious regress, the initial value is completely accounted for by contributions of the members; contributions of the members are “collected” so that the initial value is a direct function of them. For a benign regress, the initial value is not entirely accounted for by features of the members. Rather, members operate on some value

“shifted” from one stage to the next — with the result that an infinite series may have distinct stable states corresponding to different values shifted. In the vicious case, the series has a determinate output value, incompatible with adequacy. But where a series has multiple stable states, there may be no reason to deny that the series takes a state compatible with adequacy. One might desire some additional characterization of contexts which result in one functional relation rather than the other. However, I am saying something about the general structure, not the content, of infinite regress arguments. At that level, the bottom line may be just that contexts are characterized by the different functional relations.

To emphasize this point that vicious regresses may involve different functional relations, consider a couple more cases. Instead of a voltage source and resistor, suppose an individual consists of a voltage source and diode (where a diode lets current pass in only one direction).



In this case, the voltage at Q1 is equal to the maximum of the individual voltages,  $\text{Max}(V_1, V_2, V_3, V_4, V_5)$ . This works like pressures from columns of water, with check valves to prevent flow back toward the sources; then the output rises just to the height of the highest input column. Suppose we are given some such system with (i)  $Q1 = V$ ; (ii) any  $V_n$  in the series  $< V$ ; and (iii)  $Q1 = \text{Max}(V_1, V_2, \dots)$ . Then a finite  $n$  member series has  $V = \text{Max}(V_1, V_2, \dots, V_n) < V$ . But this is impossible, so no such series is finite. But similarly, in the infinite case,  $\text{Max}(V_1, V_2, \dots) < V$ ; so the premises are inconsistent. Suppose an individual consists of a voltage source and switch, with the voltage sources all fixed at some constant value  $V$ .



Then the voltage at  $Q1 = V$  iff one or more switches is closed — iff the extended disjunction,  $V[\text{Closed}(s_1), \text{Closed}(s_2), \text{Closed}(s_3), \text{Closed}(s_4), \text{Closed}(s_5)]$  is true. Say we are given some such system with (i)  $Q1 = V$ ; (ii) no  $s_n$  in the series such that  $\text{Closed}(s_n)$ ; and (iii)  $Q1 = V \leftrightarrow V[\text{Closed}(s_1), \text{Closed}(s_2)\dots]$ . Given (ii), for a finite  $n$  member series,  $V[\text{Closed}(s_1), \text{Closed}(s_2)\dots \text{Closed}(s_n)]$  is false; so by (iii),  $Q1 \neq V$ , which contradicts (i). But similarly, in the infinite case,  $V[\text{Closed}(s_1), \text{Closed}(s_2)\dots]$  is false and the premises are inconsistent.

Perhaps these last cases are too obvious, and so motivate the intuition, expressed by Geach, that there is something trivial or uninteresting about infinite regresses. Thus, e.g.,  $V[\text{Closed}(s_1), \text{Closed}(s_2)\dots]$  iff  $(\exists s_n)\text{Closed}(s_n)$ , and we are given  $\sim(\exists s_n)\text{Closed}(s_n)$ ; these conflict directly, without the rigmarole of finite and infinite series. At this stage, I note first, that there is nothing wrong about reasoning with the finite and infinite cases — it is at least one way to expose contradiction lurking in premises. Second, not all arguments of the proposed regress form are so trivial. As we have seen, reasons which result in contradiction in finite cases may or may not result in contradiction in the infinite. So the division into finite and infinite cases is not superfluous. And the difficulty with infinite regress need not be that of making no progress. There is progress in the vicious summation of currents case — only not enough; each member of the series makes some positive contribution, though the sum of contributions remains inadequate. And even in cases where there seems to be no progress from one step to the next, the situation may be relatively complex. There seems to be no progress from one step to the next in the regress of cats (or amplifiers), and similarly with the switches. Yet one is vicious and the other not. In the regress of cats, we go from one finite stage to the next without biting; but in the infinite case, all the cats may bite. In the regress of switches, we go from one finite stage to the next with zero output voltage; and voltage remains at zero in the infinite case. In general, the difficulty is making *inadequate* progress, where adequacy may be determined in relatively complex ways.

I conclude this section with some brief remarks about what has been accomplished, and the shape of argument to come. First, all our series — progress and regress, benign and vicious

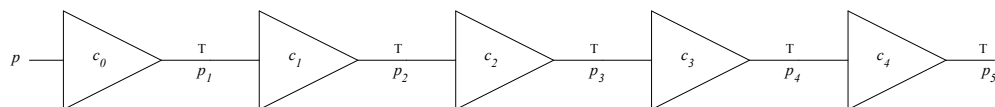
— may be described by premises in the style of Black. We begin with premises according to which there is an *R*-series whose first member is *F*; so we accept (E) and (P). And, insofar as we are in a position to reason from one stage to the next, we accept something like (G) — though we have seen cases where members do not share some constant property *F*, but rather have properties that are indexed to position in the series, varying from one member to the next. In regress cases, there is downward dependence as well. So far, then, the point is not that Black or Clark somehow go wrong in their description of infinite regresses. Rather, it is that adequacy, underlying and linking premises drive the relations described in (G). In vicious cases, these premises result also in contradiction — something that (E), (G), and (P), with downward dependence, do not by themselves do.

Second, on this account, a theory which results in vicious infinite regress is reduced to absurdity. But one might object that series I say are not vicious are, nonetheless, impossible (Craig or Clark might reason this way). Of course, I do not *prove* that series I count as progress and benign regress are possible. However, I do try to motivate or ground claims about possibility by our assumption that there is nothing the matter with infinite series as such, together with models for the different series. I thus try to motivate the suggestion that consistency for our premises tracks a larger sense of possibility. Suppose this is right, and valid regress arguments are generally characterized by premises of the sort I describe. Then debate about regresses shifts to the premises. If some situation is mistakenly described as a progress or benign regress, it is natural to object by showing how the description is mistaken and exhibiting whatever strengthened premises result in contradiction. Given that there are contradictions in the neighborhood to be had, an infinite regress argument, if valid, should include whatever premises are required to reach contradiction. However, as we will see, it remains possible to disagree about a theory's proper consequences, and so about whether a regress is vicious or benign.

### III. Philosophical Applications

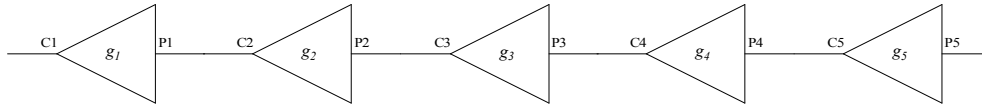
In this section, I merely scratch the surface of the arguments with which we began. In each case, variant formulations, and a variety of objections and replies, go unexamined. The point is not so much to obtain definitive results with respect to particular arguments, as to exhibit the overall shape of the theory's application to philosophical cases.

The simple truth series appears as non-vicious because it is not a regress at all; rather it is a progress — or so it seems on a “backwards” or “upwards” looking account of truth along the lines of the correspondence theory. As in the case of the mouse which bites the cats, a value at one stage is simply propagated out to the next. Thus we are given a series of stages which take as input a state or situation, and output a truth value to a proposition — where the proposition with its truth value is the input to the next. Say  $p_1$  is the proposition that  $p$  and  $p_{i+1}$  the proposition that  $p_i$  is true. Then we are given a series of stages  $c_1, c_2, \dots$  which take as input a truth value for  $p_i$  and assign a value to  $p_{i+1}$ , where by (Tp),  $p_{i+1}$  is true iff  $p_i$  is true. Stages are linked so that the output value from one stage is the input to the next. Thus, given an initial stage that makes  $p_1$  true, by (Tp),  $p_2$  is true; but this makes possible another application of (Tp); so  $p_3$  is true; and so forth. Given initial situation  $p$ , we might picture the series as follows,



If the initial situation were *not-p*, each of the outputs would be *false*. So the stages are governed by (Tp), and linked so that the output of one is the input to the next. Underlying premises about the stages, together with the way they are linked, make other values a function of the first. Thus it is a progress, not a regress. And there is no contradiction. The key to this series is that truth, on the supposed account, is a “backwards” or “upwards” looking notion. Also, depending on our view of propositions, there is nothing “uneconomical” or “unbelievable” about this series. What would be odd, is if the series were somehow to end, or to have one member true, and the next not. The situation might change on some other account of truth, but that is another story.

The hereditary series appears as a regress, but benign. In this case, we have a series of generative events, arranged so that the input P (“parent”) to one is the output C (“child”) of the next. Then we might see the series as follows,



The stages are linked so that for any  $n$ ,  $P_n = C_{n'}$ . Say generative events do not preserve humanity perfectly. Then, where the “humanity” for any output value falls between 0 and 1, and  $e_n$  is some positive or negative error value, let us say that the stages have an underlying character so that  $C_n = P_n + e_n$ . Then for a finite series,

$$\begin{aligned} C_1 &= e_1 + P_1 \\ &= e_1 + C_2 = e_1 + e_2 + P_2 \\ &\quad \vdots \\ &= e_1 + e_2 + \dots + e_n + P_n \end{aligned}$$

where equalities in the horizontal direction are by  $C_n = P_n + e_n$ , and in the vertical by  $P_n = C_{n'}$ . With the error values, there is the possibility that  $C_1 = 1$  and some  $P_n = 0$  without contradiction. But, if as in the original problem, we add that each  $e_i = 0$ , then it is inconsistent to suppose that  $C_1 = 1$  and some  $P_n \neq 1$ . Insofar as underlying and linking premises result in contradiction with adequacy, it is a regress. But when the series is infinite, contradiction evaporates: for limit value  $P_\omega$ , we have  $C_1 = P_\omega$ , but without a value given for  $P_\omega$ . So in the infinite case, underlying and linking premises do not determine a value for  $C_1$  and the regress is benign.

In contrast, the rock moved by the stick, etc. — construed as a series of train cars, is vicious. The leftmost car is accelerated by the car in front of it, which is accelerated by the one in front of it; etc. So far, then, it may seem as though acceleration is “shifted” from one car to the next and, though there is a regress, it is benign. However, with a bit of (anachronistic, as applied to Aristotle or Aquinas) physics, it is clear that the cars cannot accelerate. If the train is infinitely massive, then no amount of force makes it move — or at least our ordinary notions from physics do not apply. So suppose each car is half the size and mass of the one it pulls; then the total size and mass of the train is finite, and we still have the question about how it moves. The

acceleration of the train is equal to the total force applied to it divided by its mass,  $a = F/m$ . The cars are linked so that the force applied to an  $n$ -member train is the sum of the forces applied by each of the cars,  $F_n = \Sigma(F_1, F_2 \dots F_n)$ . But if no car is an engine, we have the underlying fact that each of the forces, and so their sum, is zero. And this is impossible if the train accelerates. So it is a regress. But similarly, in the infinite case, the sum of the forces is zero, and the train does not accelerate. So the regress is vicious. If we are tempted to see the regress as benign, I think it is because we are tempted to see it under descriptions that do not properly sum the forces from the cars.

The fundamental relation regress is more difficult — not so much because of problems with the theory of regresses, but because the theory of resemblances is itself obscure. Russell thinks that, on a strict resemblance nominalism, a relation is a resemblance only if it resembles some standard. Thus each relation requires another in order for it to count as a resemblance. But there are different ways to see this. On the one hand, we may think that a member of the series can be a resemblance iff some member of the series is a resemblance of its own intrinsic nature — iff it is a “mover” for the other members. Then stages are linked so that  $res(r_1) \leftrightarrow \forall [\text{mover}(r_1), \text{mover}(r_2) \dots]$ . Then, given the underlying premise that no member of the series is a resemblance of its own intrinsic nature, in both the finite and infinite case, there is conflict with the adequacy premise that the first member is a resemblance, so that the series is both a regress and vicious. Something like this is Russell’s view.

But it may be that each relation is simple a “child” of the next. Suppose the underlying premise that a relation is a resemblance iff it resembles the standard resemblance,  $res(r_i) \leftrightarrow r_i \simeq r^*$ , where stages are linked so that  $r_i \simeq r^* \leftrightarrow res(r_{i+1})$ . In this case,

$$\begin{aligned} res(r_1) &\leftrightarrow r_1 \simeq r^* \\ &\leftrightarrow res(r_2) \leftrightarrow r_2 \simeq r^* \text{ etc.} \end{aligned}$$

So in the finite case,  $res(r_1) \leftrightarrow res(r_2) \leftrightarrow \dots \leftrightarrow res(r_n) \leftrightarrow r_n \simeq r^*$ . In this finite case, with no resemblance after the last,  $r_n \not\simeq r^*$ ; so there is conflict with adequacy, and the series is a regress. But in the infinite case,  $res(r_1) \leftrightarrow res(r_2) \leftrightarrow \dots \leftrightarrow res(r_\omega)$  and there is nothing to prevent the limit,

and each member, of the series from being a resemblance — so the regress, on this interpretation, is benign.

In favor of Russell's interpretation, it is natural to think that if the resemblance theory is to count as a viable response to the problem of universals, a relation must be a resemblance purely as a result of the (relatively) intrinsic natures of members of the series, together with the way they are connected. If the series is benign precisely because the series has multiple stable states, it may very well not be a successful account of universals. Perhaps, though, the reason such regress arguments remain obscure is precisely because we have nowhere near so much clarity about resemblances (justification, or the like) as about relations between parents and children or train cars.

But there is yet another option. If you are six feet tall, and I am six feet tall, the way we are, individually, guarantees that we are the *same height*. The fundamental facts seem to be our individual heights, and the sameness arises because of the way we are individually. Similarly, a resemblance theorist might say that the resemblance between color patches arises because of the way they are individually. And there may be resemblances among relations but, again, because of the way they are individually — so that *resemblance* is an “internal” relation in the sense of Lewis (1986, 62ff). In this case, there may be a series of resemblances, but the series appears as a progress, not a regress, with  $res(r_i) \leftrightarrow r_i \approx r^*$ , and  $r_i \approx r^* \leftrightarrow res(r_{i+1})$  but no reason to deny that  $r_n \approx r^*$ . Quine pushes in this direction in a famous passage from his (1948). He allows that there are red houses, roses, and sunsets, but denies that they have anything in common “except as a popular and misleading manner of speaking.” That the houses and roses and sunsets are all of them red is taken as “ultimate and irreducible.” Presumably, since they are all red, they resemble one another in this respect. But they are not red because they resemble (and Quine would not want to talk as though there are *resemblances*, except as a misleading manner of speech). Rather, they resemble because they are all red. Given difficulties as above, I suspect that a viable resemblance theory would need to be developed along these lines. And similarly for other approaches to the problem

of universals. But that is well beyond the scope of this paper. All I am after is that we require clarification of the resemblance theory, before we can sensibly evaluate related regress claims.

I take it as evidence for this approach to regress arguments, that its results coincide with traditional evaluations — and even that results are indeterminate where tradition is less than clear. One might object that the theory is therefore philosophically impotent. But this would be a mistake. First, the account of regress arguments tells us what vicious infinite regress amounts to, and so how it matters; insofar as a theory which results in vicious infinite regress is inconsistent, it is reduced to absurdity in the usual way. Further, the theory tells us *what* to look for in the evaluation of regress arguments, and so guides our approach to argument evaluation; their evaluation depends on adequacy, underlying and linking premises.<sup>15</sup> Even in cases where results are not clear, the account points to ways in which a theory should be developed, before we can even sensibly evaluate regress objections and so say whether a theory is coherent; and the direction in which we ultimately develop some theory, may itself be constrained by regress concerns. And this, I take it, is progress indeed.<sup>16</sup>

## Notes

<sup>1</sup>But see e.g. W. L. Craig (1979) and other places, who grants the point about mathematics, but develops and defends traditional arguments against the actual infinite. I am inclined to think these arguments fail. *Pace* Craig, difficulties raised by ancient and medieval philosophers seem resolved with the rise of contemporary mathematics. See p. 16 below.

<sup>2</sup>Aquinas, *Summa Theologiae* I.2.3, trans. Anton Pegis. See, e.g., Aristotle, *Metaphysics* 994a1-19. In Aquinas, the distinction between this and the previous case is between *accidental* and *per se* regresses. For discussion, see (Day 1987) and (Brown 1966).

<sup>3</sup>Strangely, Russell seems not to have seen the possibility of application against his own view. For general treatment, see (Armstrong 1974).

<sup>4</sup>Even more recently, (Nolan 2001) argues that vicious infinite regresses are characterized by various theoretical difficulties, including ontological extravagance. Of course, I do not *deny* that even benign regresses and series that are not regresses at all may suffer from such difficulties. My main response to Nolan is the account on which there is more than this to *vicious* infinite regress.

<sup>5</sup>The *ancestral*  $*R$  of a binary relation  $R$ , is the relation such that (i) for any  $x$  and  $y$ , if  $Rxy$  then  $*Rxy$ ; (ii) for any  $x, y$  and  $z$ , if  $*Rxy$  and  $*Ryz$ , then  $*Rxz$ ; and (iii) for no other  $x$  and  $y$  is it the case that  $*Rxy$ .

<sup>6</sup>Let  $A$  be the set of all  $RF$ -series, and consider some  $B \subseteq A$  such that for any  $C, D \in B$  either  $C \subseteq D$  or  $D \subseteq C$ . For an application of the axiom of choice (in the form of Zorn's lemma), we require that  $\bigcup B \in A$ . If  $\langle n, c \rangle$  and  $\langle n, d \rangle$  are in  $\bigcup B$ , then  $\langle n, c \rangle \in C \in B$  and  $\langle n, d \rangle \in D \in B$ ; but  $C \subseteq D$  or  $D \subseteq C$ ; either way, both  $\langle n, c \rangle$  and  $\langle n, d \rangle$  are members of some one function; so  $c = d$ ; so  $\bigcup B$  is a function. The domain of a union of functions is the union of their domains, and a union of initial segments of the integers is an initial segment of the integers; so the domain of  $\bigcup B$  is an initial segment of the integers; so  $\bigcup B$  is a sequence. If  $c_n$  is in  $\bigcup B$  then  $c_n \in C \in B$ ; since the

domain of  $C$  is an initial segment of the integers,  $c_n \in C$ ; since  $C$  is an  $R$ -series,  $Rc_n c_n$ ; so  $\bigcup B$  is an  $R$ -series. Finally, since each member of  $\bigcup B$  is a member of an  $RF$ -series, each member of  $\bigcup B$  is  $F$ ; so  $\bigcup B$  is an  $RF$ -series. So  $\bigcup B \in A$ . So by Zorn's lemma, there is an  $RF$  series  $M \in A$  which is not a subset of any other member of  $A$ . Suppose  $M$  is finite; then for some  $i$ ,  $M = \langle m_1, m_2, \dots, m_i \rangle$ . Since each member of  $M$  is  $F$ ,  $m_1$  is  $F$ ; so by (G), there is an  $a$  such that  $Fa$  and  $Rm_1 a$ ; so  $M \cup \{i+1, a\}$  is an  $RF$  series; so there is an  $RF$ -series  $\langle m_1, m_2, \dots, m_i, m_{i+1} \rangle$  of which  $M$  is a subset. This contradicts the maximality of  $M$ ; reject the assumption:  $M$  is infinite.

<sup>7</sup>But on Black's terms, 'vicious' in 'vicious infinite regress' is redundant. He sets things up so that the notion of an infinite regress itself has 'dyslogistic' force (115-124).

<sup>8</sup>In such cases we might have an independent premise like Black's (F); see also Nolan's discussion of regresses with known finite domains (2001, 531-32). In these cases, the *regresses* might or might not be *vicious* in the sense discussed below.

<sup>9</sup>For the main argument above, it is sufficient simply to accept the claims about arithmetic. However, this result should be familiar to philosophers from Zeno's paradox of dichotomy. If one goes half some distance, half the remaining, etc., after any  $n$  steps,  $1/2^n$  the original distance remains. More formally, arguing by induction,  $\Sigma(1/2^1) = 1/2 = 1 - 1/2^1$ ; suppose  $\Sigma(1/2^1 \dots 1/2^k) = 1 - 1/2^k$ ; then  $\Sigma(1/2^1 \dots 1/2^k, 1/2^{k+1}) = 1 - 1/2^k + 1/2^{k+1} = 1 - 2/2^{k+1} + 1/2^{k+1} = 1 - 1/2^{k+1}$ ; so for arbitrary  $n$ ,  $\Sigma(1/2^1 \dots 1/2^n) = 1 - 1/2^n$ .

<sup>10</sup>See, e.g., (Craig 1979, 84-86). Craig's cases manipulate occupants of the rooms, and books on the shelves of an infinite library, not soap. I have found the soap example effective because of the (seemingly) absurd potential for profit. But the point is the same.

<sup>11</sup>For another case with the same structure, consider a series of weights placed upon a scale. Then the total force on the scale is equal to the sum of the forces from the weights, and the system works like the summation of currents.

<sup>12</sup>There is room for caution on this point: On the standard theory for real numbers, an infinite sum *is* the series of its partial sums, where the series of sums *is*, or designates, a real number. So it is not clear how or whether the sum counts as a straightforward *total* of the infinitely

many members. Consider, e.g., “conditionally” convergent series as,  $1 - 1/2 + 1/3 - 1/4 + 1/5...$  and  $1 + 1/3 - 1/2 + 1/5 + 1/7 - 1/4...$  which have all the same members, but different limits and so different sums (see, e.g., Knopp 1928, 102-103 and 139ff). Such concerns are immaterial to the overall view illustrated by this example, where the important point is just that the series has some definite output value. Still, in the above case, it seems reasonable to think that the value is one. Setting aside the point that current is discrete at the atomic level, the current at Q1 has some real value; it can not be a value less than one since, at some stage, the current must exceed any such value; it can not be a value more than one, since one is the limit of the series. So the value is one.

<sup>13</sup>The above example of a regress takes the infinity of the series of amplifiers as a *premise*. The progression is not from one amplifier to the next, but from the value of one output, to the value of the next. If the series were finite, the values would progress through as many amplifiers as there were and stop. In contrast, the regress of amplifiers does not take the infinity of the series as a premise; the existence of members is driven by contradiction in every finite case. But this is not so if  $Q1 = L$ . Since  $Q1$  for every finite series  $= L$ , from  $Q1 = L$  with (ii) and (iii), there is no requirement that there be *any* other amplifier. But, however many amplifiers there are, a first value of  $L$  depends on a value of  $L$  at the next, all the way to the last, if there is a last. On the account I offer, it is a regress insofar as the first value depends on the rest. It is benign insofar as there is no contradiction in the infinite case. A vicious regress *requires* conflict between (i) and the results of (ii) and (iii).

<sup>14</sup>For this, we might imagine analog multiplication devices such that  $Qn = V_n * Dn$ , arranged in parallel as for the summation of currents. Then, for the infinite case,  $Q1 = \Pi(V_1 * V_2 * ...)$  and with underlying premises about the values of  $V_1, V_2, ...$ , there is potential for conflict with adequacy.

<sup>15</sup>So, in a recent discussion, (Gillett 2003; Klein 2003) Carl Gillett argues that Peter Klein’s “infinetism” as a theory of epistemic justification is subject to regress objections – on grounds something like downward dependence of justification. Klein responds in part, that series characterized by downward dependence are not vicious – using as his example a series like the

rock being moved by the stick, etc. But we have held that (i) downward dependence is not by itself sufficient for vicious regress; and (ii) the series of train cars, at least, is in fact vicious. So far, then, neither the attack nor the defense succeeds. (But in note 5 on p. 713, Gillett suggests that downward dependence forces only the existence of an actually infinite series, so that “all the problems surrounding the actual infinite consequently dog infinitism.” This suggests a premise like Black’s – that there is no infinite series of the appropriate sort. But Gillett does not say what the problems are supposed to be; and we have held that premises in the style of Black’s themselves follow from reasons driving vicious regress.)

<sup>16</sup>A version of this paper was presented to the philosophy department at California State University, San Bernardino. I thank participants in that discussion, along with Rose Roy, John Sisko, an anonymous referee, and especially Matthew Davidson and Darcy Otto for many helpful comments.

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