

Validity and Soundness

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In this short paper, I introduce two central notions for argument evaluation. The presentation is completely informal. It is possible to develop formal methods for working with validity and soundness, but it is also possible to apply the informal notions directly to problems in philosophy and beyond. In either case, it is important to understand the basic notions, in order to understand *what* is accomplished in reasoning. Exercises are included, with answers to selected exercises at the end.

For our purposes, an argument is not a shouting match of the sort that might come to blows. Rather,

AR An *argument* is some sentences, one of which (the *conclusion*) is taken to be supported by the remaining sentences (the *premises*).

So some sentences are an argument depending on whether premises are taken to support a conclusion. Such support is often indicated by words or phrases of the sort, ‘so’, ‘it follows’, ‘therefore’, or the like. We will typically indicate the division by a simple line between premises and conclusion. Roughly, an argument is good if premises do what they are taken to do, if they actually support the conclusion. An argument is bad if they do not accomplish what they are taken to do, if they do not actually support the conclusion.

Logical validity and soundness correspond to different ways an argument can go wrong. Consider the following two arguments:

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	Only citizens can vote		All citizens can vote
(A)	<u>Hannah is a citizen</u>	(B)	<u>Hannah is a citizen</u>
	Hannah can vote		Hannah can vote

The line divides premises from conclusion, indicating that the premises are supposed to support the conclusion. Thus these are arguments. But the arguments go wrong in different ways. The premises of argument (A) are true; as a matter of fact, only citizens can vote, and Hannah (my daughter) is a citizen. But she cannot vote; she is not old enough. So the conclusion is false. Thus, in argument (A), the relation between the premises and the conclusion is defective. Even though the premises are true, there is no guarantee that the conclusion is true as well. We will say that this argument is *logically invalid*. In contrast, argument (B) is logically valid. If its premises were true, the conclusion would be true as well. So the *relation* between the premises and conclusion is not defective. The problem with this argument is that the premises are not true — not all citizens can vote. So argument (B) is defective, but in a different way. We will say that it is *logically unsound*.

In this short paper, I define and explain these notions of logical validity and soundness. I begin with some preliminary notions, then turn to official definitions of logical validity and soundness, and finally to some consequences of the definitions.

1 Consistent Stories

Given a certain notion of a *possible* or *consistent* story, it is easy to state definitions for logical validity and soundness. So I begin by identifying the kind of stories that matter. Then we will be in a position to state the definitions, and apply them in some simple cases.

Let us begin with the observation that there are different sorts of possibility. Consider, say, “Hannah could make it in the WNBA.” This seems true. She is reasonably athletic, and if she were to devote herself to basketball over the next few years, she might very well make it in the WNBA. But wait! Hannah is only a kid — she rarely gets the ball even to the rim from the top of the key — so there is no way she could make it in the WNBA. So she both could and could not make it. But this cannot be right! What is going on? Here is a plausible explanation: Different sorts of possibility are involved. When we hold fixed current abilities, we are inclined to say there is no way she could make it. When we hold fixed only general physical characteristics,

and allow for development, it is natural to say that she might. The scope of what is possible varies with whatever constraints are in play. The weaker the constraints, the broader the range of what is possible.

The sort of possibility we are interested in is *very* broad, and constraints are correspondingly weak. We will allow that a story is *possible* or *consistent* so long as it involves no *internal* contradiction. A story is impossible when it collapses from within. For this it may help to think about the way you respond to ordinary fiction. Consider, say, *Bill and Ted's Excellent Adventure* (set and partly filmed locally for me in San Dimas, CA). Bill and Ted travel through time in a modified phone booth collecting historical figures for a history project. Taken seriously, this is bizarre, and it is particularly outlandish to think that a *phone booth* should travel through time. But the movie does not so far contradict itself. So you go along. So far, then, so good (excellent).

But, late in the movie, Bill and Ted have a problem breaking the historical figures out of jail. So they decide today to go back in time tomorrow to set up a diversion that will go off in the present. The diversion goes off as planned, and the day is saved. Somehow, then, as often happens in these films, the past depends on the future, at the same time as the future depends on the past. This, rather than the time travel itself, generates an internal conflict. The movie makes it the case that you cannot have today apart from tomorrow, and cannot have tomorrow apart from today. Perhaps today and tomorrow have always been repeating in an eternal loop. But, according to the movie, there were times before today and after tomorrow. So the movie faces *internal* collapse. Notice: the objection does not have *anything* to do with the way things actually are — with the nature of actual phone booths and the like; it has rather to do with the way the movie hangs together internally — it makes it impossible for today to happen without tomorrow, and for tomorrow to happen without today.¹ Similarly, we want to ask whether stories hold together *internally*. If a story holds together internally, it counts for our purposes, as consistent and possible. If a story does not hold together, it is not consistent or possible.

¹In more consistent cases of time travel (in the movies) time seems to move in a sort of 'Z' so that after yesterday and today, there is *another* yesterday and *another* today. So time does not return to the very point at which it first turns back. In the trouble cases, however, time seems to move in a sort of "loop" so that a point on the path to today (this very day) goes through tomorrow. With this in mind, it is interesting to think about say, the *Terminator* and *Back to the Future* movies and, maybe more consistent, *Groundhog Day*. Even if I am wrong, and *Bill and Ted* is internally consistent, the overall point should be clear. And it should be clear that I am not saying anything serious about time travel.

In some cases, then, stories may be consistent with things we know are true in the real world. Thus perhaps I come home, notice that Hannah is not in her room, and imagine that she is out back shooting baskets. There is nothing inconsistent about this. But stories may remain consistent though they do not fit with what we know to be true in the real world. Here are cases of phone booths traveling through time and the like. Stories become inconsistent when they collapse internally — as when today both can and cannot happen apart from tomorrow.

As with a movie or novel, we can say that different things are true or false in our stories. In *Bill and Ted's Excellent Adventure* it is true that Bill and Ted travel through time in a phone booth, but false that they go through time in a DeLorean (as in the *Back to the Future* films). In the real world, of course, it is false that phone booths go through time, and false that DeLoreans go through time. Officially, a complete story is always *maximal* in the sense that *any* sentence is either true or false in it. A story is *inconsistent* when it makes some sentence both true and false. Since, ordinarily, we do not describe every detail of what is true and what is false when we tell a story, what we tell is only part of a maximal story. In practice, however, it will be sufficient for us merely to give or fill in whatever details are relevant in a particular context.

But there are a couple of cases where we cannot say when sentences are true or false in a story. The first is when stories we tell do not fill in relevant details. In *The Wizard of Oz*, it is true that Dorothy wears red shoes. But neither the movie nor the book have anything to say about whether her shoes include Odor-Eaters. By themselves, then, neither the book nor the movie give us enough information to tell whether “The red shoes include Odor-Eaters” is true or false in the story. Similarly, there is a problem when stories are inconsistent. Suppose according to some story,

- (a) All dogs can fly
- (b) Fido is a dog
- (c) Fido cannot fly

Given (a), all dogs fly; but from (b) and (c), it seems that not all dogs fly. Given (b), Fido is a dog; but from (a) and (c) it seems that Fido is not a dog. Given (c), Fido cannot fly; but from (a) and (b) it seems that Fido can fly. The problem is not that inconsistent stories say too little, but rather that they say too much. When a story is inconsistent, we will simply refuse to say

that it makes any sentence (simply) true or false.²

Consider some examples: (a) The true story, “Everything is as it actually is.” Since no contradiction is actually true, this story involves no contradiction; so it is internally consistent and possible.

(b) “All dogs can fly: over the years, dogs have developed extraordinarily large and muscular ears; with these ears, dogs can fly.” It is bizarre, but not obviously inconsistent. If we allow the consistency of stories according to which monkeys fly, as in *The Wizard of Oz*, or elephants fly, as in *Dumbo*, then we should allow that this story is consistent as well.

(c) “All dogs can fly, but my dog Fido cannot; Fido’s ear was injured while he was chasing a helicopter, and he cannot fly.” This is *not* internally consistent. If all dogs can fly and Fido is a dog, then Fido can fly. You might think that Fido remains a flying sort of thing. In evaluating internal consistency, however, we require that *meanings remain the same*: If “can fly” means just “is a flying sort of thing,” then the story falls apart insofar as it says both that Fido is and is not that sort of thing; if “can fly” means “is himself able to fly,” then the story falls apart insofar as it says that Fido himself both is and is not able to fly. So long as “can fly” means the same in each use, the story is sure to fall apart insofar as it says both that Fido is and is not that sort of thing.

(d) “Germany won WWII; the United States never entered the war; after a long and gallant struggle, England and the rest of Europe surrendered.” It did not happen; but the story does not contradict itself. For our purposes, then it counts as possible.

(e) “ $1 + 1 = 3$; the numerals ‘2’ and ‘3’ are switched (‘1’, ‘3’, ‘2’, ‘4’, ‘5’, ‘6’, ‘7’...); so that taking one thing and one thing results in three things.” This story does not hang together. Of course numerals can be switched; but switching numerals does not make one thing and one thing three things! We tell stories in our own language (imagine that you are describing a foreign-language film in English). According to the story, people can say correctly

Some authors prefer talk of “possible worlds,” “possible situations” or the like to that of consistent stories. It is conceptually simpler to stick with stories, as I have, than to have situations and distinct descriptions of them. However, it is worth recognizing that our consistent stories are or describe possible situations, so that the one notion matches up directly with the others.

²The intuitive picture developed above should be sufficient for our purposes. However, we are on the verge of vexed issues. For further discussion, you may want to check out the vast literature on “possible worlds.” Contributions of my own include, “Worlds and Modality,” and “In Defense of Linguistic Ersatzism.”

' $1 + 1 = 3$ ', but this does not make it the case that $1 + 1 = 3$. Compare a language like English except that 'fly' means 'bark'; and consider a movie where dogs are ordinary, but people correctly assert, in this language, "dogs fly": it would be wrong to say, in *English*, that this is a movie in which *dogs fly*. And, similarly, we have not told a story where $1 + 1 = 3$.

- E1.1. Say whether each of the following stories is internally consistent or inconsistent. In either case, explain why.
- a. Smoking cigarettes greatly increases the risk of lung cancer, although most people who smoke cigarettes do not get lung cancer.
 - b. Joe is taller than Mary, but Mary is taller than Joe.
 - c. Abortion is always morally wrong, though abortion is morally right in order to save a woman's life.
 - d. Mildred is Dr. Saunders's daughter, although Dr. Saunders is not Mildred's father.
 - e. No rabbits are nearsighted, though some rabbits wear glasses.
 - f. Ray got an 'A' on the final exam in both Phil 200 and Phil 192. But he got a 'C' on the final exam in Phil 192.
 - g. Bill Clinton was never president of the United States, although Hillary is president right now.
 - h. Egypt, with about 100 million people is the most populous country in Africa, and Africa contains the most populous country in the world. But the United States has over 200 million people.
 - i. The death star is a weapon more powerful than that in any galaxy, though there is, in a galaxy far far away, a weapon more powerful than it.
 - j. Luke and the rebellion valiantly battled the evil empire, only to be defeated. The story ends there.
- E1.2. For each of the following sentences, (i) say whether it is true or false in the real world and then (ii) say if you can whether it is true or false according to the accompanying story. In each case, explain your answers.

- a. Sentence: Aaron Burr was never a president of the United States.
Story: Aaron Burr was the first president of the United States, however he turned traitor and was impeached and then executed.
- b. Sentence: In 2006, there were still buffalo.
Story: A thundering herd of buffalo overran Phoenix Arizona in early 2006. The city no longer exists.
- c. Sentence: After overrunning Phoenix, the buffalo did in Newark, New Jersey.
Story: A thundering herd of buffalo overran Phoenix Arizona in early 2006. The city no longer exists.
- d. Sentence: There has been an all-out nuclear war.
Story: After the all-out nuclear war, John Connor organized resistance against the machines — who had taken over the world for themselves.
- e. Sentence: Jack Nicholson has swum the Atlantic.
Story: No human being has swum the Atlantic. Jack Nicholson and Bill Clinton and you are all human beings, and at least one of you swam all the way across!
- f. Sentence: Some people have died as a result of nuclear explosions.
Story: As a result of a nuclear blast that wiped out most of this continent, you have been dead for over a year.
- g. Sentence: Your instructor is not a human being.
Story: Aliens have never made it to this country. However, without a doubt, your instructor is from another planet.
- h. Sentence: Lassie is both a television and movie star.
Story: Dogs have super-big ears and have learned to fly. Indeed, all dogs can fly. Among the many dogs are Lassie and Rin Tin Tin.
- i. Sentence: The Yugo is the most expensive car in the world.
Story: Jaguar and Rolls Royce are expensive cars. But the Yugo is more expensive than either of them.

j. Sentence: Lassie is a bird who has learned to fly.

Story: Dogs have super-big ears and have learned to fly. Indeed, all dogs can fly. Among the many dogs are Lassie and Rin Tin Tin.

2 The Definitions

The definition of logical validity depends on what is true and false in consistent stories. The definition of soundness builds directly on the definition of validity. Note: in offering these definitions, I *stipulate* the way the terms are to be used; there is no attempt to say how they are used in ordinary conversation; rather, we say what they will mean for us in this context.

LV An argument is *logically valid* if and only if (iff) there is no consistent story in which all the premises are true and the conclusion is false.

LS An argument is *logically sound* iff it is logically valid and all of its premises are true in the real world.

Logical (deductive) validity and soundness are to be distinguished from *inductive* validity and soundness or success. For the inductive case, it is natural to focus on the *plausibility* or the *probability* of stories — where an argument is relatively strong when stories that make the premises true and conclusion false are relatively implausible. Logical (deductive) validity and soundness are thus a sort of limiting case, where stories that make premises true and conclusion false are not merely implausible, but impossible. In a deductive argument, conclusions are supposed to be *guaranteed*; in an inductive argument, conclusions are merely supposed to be made probable or plausible. For mathematical logic, we set the inductive case to the side, and focus on the deductive.

2.1 Invalidity

If an argument is logically valid, there is no consistent story that makes the premises true and conclusion false. So, to show that an argument is invalid, it is enough to *produce* even one consistent story that makes premises true and conclusion false. Perhaps there are stories that result in other combinations of true and false for the premises and conclusion; this does not matter for the definition. However, if there is even one story that makes premises true and

conclusion false then, by definition, the argument is not logically valid — and if it is not valid, by definition, it is not logically sound. We can work through this reasoning by means of a simple *invalidity test*. Given an argument, this test has the following four stages.

- IT
- a. List the premises and negation of the conclusion.
 - b. Produce a consistent story in which the statements from (a) are all true.
 - c. Apply the definition of validity.
 - d. Apply the definition of soundness.

We begin by considering what needs to be done to show invalidity. Then we do it. Finally we apply the definitions to get the results. For a simple example, consider the following argument,

Eating Brussels sprouts results in good health

(C) Ophilia has good health

Ophilia has been eating brussels sprouts

The definition of validity has to do with whether there are consistent stories in which the premises are true and the conclusion false. Thus, in the first stage, we simply write down what would be the case in story of this sort.

- | | |
|--|---|
| a. List premises and negation of conclusion. | In any story with premises true and conclusion false, |
| | (1) Eating brussels sprouts results in good health |
| | (2) Ophilia has good health |
| | (3) Ophilia has not been eating brussels sprouts |

Observe that the conclusion is reversed! At this stage we are not giving an argument. We rather merely list what is the case when the premises are true and conclusion false. Thus there is no line between premises and the last line, insofar as there is no suggestion of support. It is easy enough to repeat the premises. Then we say what is required for the conclusion to be *false*. Thus, “Ophilia has been eating brussels sprouts” is false if Ophilia has not been eating brussels sprouts. I return to this point below, but that is enough for now.

An argument is invalid if there is even one consistent story that makes the premises true and the conclusion false. Thus, to show invalidity, it is enough *produce* a consistent story that makes the premises true and conclusion false.

- b. Produce a consistent story in which the statements from (a) are all true. Story: Eating brussels sprouts results in good health, but eating spinach does so as well; Ophilia is in good health but has been eating spinach, not brussels sprouts.

For each of the statements listed in (a), we satisfy (1) insofar as eating brussels sprouts results in good health, (2) since Ophilia is in good health, and (3) since Ophilia has not been eating brussels sprouts. The story *explains* how she manages to maintain her health without eating brussels sprouts, and so the consistency of (1) - (3) together. The story does not have to be true — and, of course, many different stories will do. All that matters is that there is a *consistent* story in which the premises of the original argument are true, and the conclusion is false.

Producing a story that makes the premises true and conclusion false is the creative part. What remains is to apply the definitions of validity and soundness. If there is a consistent story where the premises are true and the conclusion is false, then by **LV** the argument is not logically valid.

- c. Apply the definition of validity. This is a consistent story that makes the premises true and the conclusion false; thus, by definition, the argument is not logically valid.

By **LS**, for an argument to be sound, it must have its premises true in the real world *and* be logically valid. Thus if an argument fails to be logically valid, it automatically fails to be logically sound.

- d. Apply the definition of soundness. Since the argument is not logically valid, by definition, it is not logically sound.

Given an argument, the definition of validity depends on stories that make the premises true and the conclusion false. Thus, in step (a) we simply list claims required of any such story. To show invalidity, in step (b), we produce a consistent story that satisfies each of those claims. Then in steps (c) and (d) we apply the definitions to get the final results; for invalidity, these last steps are the same in every case.

It may be helpful to think of stories as a sort of “wedge” to pry the premises of an argument off its conclusion. We pry the premises off the conclusion if there is a consistent way to make the premises true and the conclusion not. If it is possible to insert such a wedge between the premises and conclusion, then a

defect is exposed in the way premises are connected to the conclusion. Observe that the flexibility allowed in consistent stories (with flying dogs and the like) corresponds directly to the strength of connections required. If connections are sufficient to resist all such attempts to wedge the premises off the conclusion, they are significant indeed.

Here is another example of our method. Though the argument may seem on its face not to be a very good one, we can expose its failure by our methods — in fact, our method may formalize or make rigorous a way you very naturally think about cases of this sort. Here is the argument,

- (D) $\frac{\text{I shall run for president}}{\text{I will be one of the most powerful men on earth}}$

To show that the argument is invalid, we turn to our standard procedure.

- a. In any story with the premise true and conclusion false,
 1. I shall run for president
 2. I will not be one of the most powerful men on earth
- b. Story: I do run for president, but get no financing and gain no votes; I lose the election. In the process, I lose my job as a professor and end up begging for scraps outside a Domino's Pizza restaurant. I fail to become one of the most powerful men on earth.
- c. This is a consistent story that makes the premise true and the conclusion false; thus, by definition, the argument is not logically valid.
- d. Since the argument is not logically valid, by definition, it is not logically sound.

This story forces a wedge between the premise and the conclusion. Thus we use the definition of validity to explain why the conclusion does not properly follow from the premises.

If you follow this method for showing invalidity, the place where you are most likely to go wrong is stage (b), telling stories where the premises are true and the conclusion false. Be sure that your story is consistent, and that it verifies *each* of the claims from stage (a). If you do this, you will be fine.

- E2.1. Use our invalidity test to show that each of the following arguments is not logically valid, and so not logically sound. Understand terms in their most natural sense.

- a. If Joe works hard, then he will get an ‘A’
 Joe will get an ‘A’
 Joe works hard
- b. Harry had his heart ripped out by a government agent
 Harry is dead
- c. Everyone who loves logic is happy
 Jane does not love logic
 Jane is not happy
- d. Our car will not run unless it has gasoline
 Our car has gasoline
 Our car will run
- e. Only citizens can vote
 Hannah is a citizen
 Hannah can vote

2.2 Validity

For a given argument, if you cannot find a story that makes the premises true and conclusion false, you may begin to suspect that it is valid. However, mere failure to demonstrate invalidity does not demonstrate validity — for all we know, there might be some tricky story we have not thought of yet. So, to show validity, we need another approach. If we could show that every story which makes the premises true and conclusion false is *inconsistent*, then we could be sure that no *consistent* story makes the premises true and conclusion false — and so we could conclude that the argument is valid. Again, we can work through this by means of a procedure, this time a *validity test*.

- VT
- a. List the premises and negation of the conclusion.
 - b. Expose the inconsistency of such a story.
 - c. Apply the definition of validity.
 - d. Apply the definition of soundness.

In this case, we begin in just the same way. The key difference arises at stage (b). For an example, consider this sample argument.

No car is a person

- (E) My mother is a person
 My mother is not a car

Since LV has to do with stories where the premises are true and the conclusion false, as before we begin by listing the premises together with the negation of the conclusion.

- a. List premises and negation of conclusion. In any story with premises true and conclusion false,
- (1) No car is a person
- (2) My mother is a person
- (3) My mother is a car

Any story where “My mother is not a car” is false, is one where my mother is a car (perhaps along the lines of the much reviled 1965 TV series, “My Mother the Car.”).

For invalidity, we would produce a consistent story in which (1) - (3) are all true. In this case, to show that the argument is valid, we show that this *cannot* be done. That is, we show that no story that makes each of (1) - (3) true is consistent.

- b. Expose the inconsistency of such a story. In any such story,
- Given (1) and (3),
- (4) My mother is not a person
- Given (2) and (4),
- (5) My mother is and is not a person

The reasoning should be clear if you focus *just on the specified lines*. Given (1) and (3), if no car is a person and my mother is a car, then my mother is not a person. But then my mother is a person from (2) and not a person from (4). So we have our goal: any story with (1) - (3) as members contradicts itself and therefore is not consistent. Observe that we could have reached this result in other ways. For example, we might have reasoned from (1) and (2) that (4'), my mother is not a car; and then from (3) and (4') to the result that (5') my mother is and is not a car. Either way, an inconsistency is exposed. Thus, as before, there are different options for this creative part.

Now we are ready to apply the definitions of logical validity and soundness. First,

- c. Apply the definition of validity. So any story with premises true and conclusion false is inconsistent and, by definition, the argument is logically valid.

Our reasoning shows that *any story with true premises and false conclusion is inconsistent*. It is worth noting that this does not quite match the wording from LV, on which an argument is valid just in case there is *no consistent story in which the premises are true and the conclusion false*. But these amount to the same thing. To see this, observe that either says the consistent stories and stories with true premises and false conclusion are *not the same*. In the one case we say this by observing that stories with true premises and false conclusion are not consistent. In the other case, we say this by observing that consistent stories are not ones with true premises and false conclusion. But it is the same underlying point in either case. If you have trouble seeing this, compare, “all criminals are unhappy” with “no happy person is a criminal.” Again, the underlying point will be that the happy people and the criminals are *distinct*. Given this, the result from (b) applies directly to the definition LV and we conclude that the argument is logically valid.

Given that the argument is logically valid, LS makes logical soundness depend on whether the premises are true in the real world. Suppose we think the premises of our argument are in fact true. Then,

- d. Apply the definition of soundness. Since in the real world no car is a person and my mother is a person, all the premises are true; so by definition, it is logically sound.

Observe that LS requires for logical soundness that an argument is logically valid and that its *premises* are true in the real world. Thus we are no longer thinking about merely possible stories! And we do not say anything at this stage about claims other than the premises of the original argument! Thus we do not make any claim about the truth or falsity of the conclusion, “my mother is not a car.” Rather, the observations have entirely to do with the two premises, “no car is a person” and “my mother is a person.” When an argument is valid and the premises are true in the real world, by LS, it is logically sound. But it will not always be the case that a valid argument has true premises. Say “My Mother the Car” is in fact a documentary — and therefore a true account of some car that is a person. Then some cars are persons and the first premise is false; so you would have to respond as follows,

- d. Since in the real world some cars are persons, not all the premises are true. So, though the argument is logically valid, by definition it is not logically sound.

Another option is that you are in doubt about reincarnation into cars, and in particular about whether some cars are persons. In this case you might respond as follows,

- d. Although in the real world my mother is a person, I cannot say whether no car is a person; so I cannot say whether all the premises are true. So although the argument is logically valid, I cannot say whether it is logically sound.

So given validity there are three options: (i) You are in a position to identify all of the premises as true in the real world. In this case, you should do so, and apply the definition for the conclusion that the argument is logically sound. (ii) You are in a position to say that at least one of the premises is false in the real world. In this case, you should do so, and apply the definition for the conclusion that the argument is not logically sound. (iii) You cannot identify any premise as false, but neither can you identify them all as true. In this case, you should explain the situation and apply the definition for the result that you are not in a position to say whether the argument is logically sound.

Again, given an argument we say in step (a) what would be the case in any story that makes the premises true and the conclusion false. Then, at step (b), instead of finding a consistent story in which the premises are true and conclusion false, we show that there is no such thing. Steps (c) and (d) apply the definitions for the final results. Observe that only one method can be correctly applied in a given case! If we can produce a consistent story according to which the premises are true and the conclusion is false, then it is not the case that no consistent story makes the premises true and the conclusion false. Similarly, if no consistent story makes the premises true and the conclusion false, then we will not be able to produce a consistent story that makes the premises true and the conclusion false.

In this case, the most difficult steps are (a) and (b), where we say what is the case in every story that makes the premises true and the conclusion false. For an example, consider the following argument.

- Some collies can fly
- (F) All collies are dogs
- All dogs can fly

It is invalid. We can easily tell a story that makes the premises true and the conclusion false — say one where Lassie is a collie who can fly, but otherwise things are as usual. Suppose, however, that we proceed with the validity test as follows,

- a. In any story with premises true and conclusion false,
 - (1) Some collies can fly
 - (2) All collies are dogs
 - (3) No dogs can fly
- b. In any such story,
 - Given (1) and (2),
 - (4) Some dogs can fly
 - Given (3) and (4),
 - (5) Some dogs can and cannot fly
- c. So any story with premises true and conclusion false is inconsistent, and by definition the argument is logically valid.
- d. Since in the real world no collies can fly, not all the premises are true. So, though the argument is logically valid, by definition it is not logically sound.

The reasoning at (b), (c) and (d) is correct. Any story with (1) - (3) is inconsistent. But something is wrong. (Can you see what?) There is a mistake at (a): It is not the case that every story that makes the premises true and conclusion false makes (3) true. The negation of “All dogs can fly” is not “No dogs can fly,” but rather, “Not all dogs can fly” (“Some dogs cannot fly”). All it takes to falsify the claim that all dogs fly, is one dog that does not. Thus, for example, all it takes to falsify the claim that everyone will get an ‘A’ is one person who does not (on this, see the extended discussion on p. 17).

We have indeed shown that every story of a certain sort is inconsistent, but have not shown that every story which makes the premises true and conclusion false is inconsistent. In fact, as we have seen, there are consistent stories that make the premises true and conclusion false. Similarly, in step (c) it is easy to get confused if you consider too much information at once. Ordinarily, if you focus on sentences singly or in pairs, it will be clear what must be the case in

Negation and Quantity

In general you want to be careful about negations. To negate any claim \mathcal{P} it is always correct to write simply, *it is not the case that \mathcal{P}* . You may choose to do this for conclusions in the first step of our procedures. At some stage, however, you will need to understand what the negation comes to. We have chosen to offer interpreted versions in the text. It is easy enough to see that,

My mother is a car and My mother is not a car

negate one another. However, there are cases where caution is required. This is particularly the case where quantity terms are involved.

In the first step of our procedures, we say what is the case in *any* story where the premises are true and the conclusion is false. The negation of a claim states what is *required* for falsity, and so meets this condition. If I say there are at least ten apples in the basket, my claim is of course false if there are only three. But not every story where my claim is false is one in which there are three apples. Rather, my claim is false just in case there are less than ten. *Any* story in which there are less than ten makes my claim false.

A related problem arises with other quantity terms. To bring this out, consider grade examples: First, if a professor says, “everyone will not get an ‘A’,” she says something disastrous. To deny it, all you need is one person to get an ‘A’. In contrast, if she says, “someone will not get an ‘A’” (“not everyone will get an ‘A’”), she says only what you expect from the start. To deny it, you need that everyone will get an ‘A’. Thus the following pairs negate one another.

Everybody will get an ‘A’ and Somebody will not get an ‘A’
Somebody will get an ‘A’ and Everybody will not get an ‘A’

A sort of rule is that pushing or pulling ‘not’ past ‘all’ or ‘some’ flips one to the other. But it is difficult to make rules for arbitrary quantity terms. So it is best just to think about what you are saying, perhaps with reference to examples like these. Thus the following also are negations of one another.

Somebody will get an ‘A’ and Nobody will get an ‘A’
Only jocks will get an ‘A’ and Some non-jock will get an ‘A’

The first works because “nobody will get an ‘A’” is just like “everybody will not get an ‘A’,” so the first pair reduces to the parallel one above. In the second case, everything turns on whether a non-jock gets an ‘A’: if none does, then only jocks will get an ‘A’; if one or more do, then some non-jock does get an ‘A’.

every story including those sentences. It does not matter which sentences you consider in what order, so long as you reach a contradiction in the end.

So far, we have seen our procedures applied in contexts where it is given ahead of time whether an argument is valid or invalid. And some exercises have been this way too. But not all situations are so simple. In the ordinary case, it is not given whether an argument is valid or invalid. In this case, there is no magic way to say ahead of time which of our two tests, **IT** or **VT** applies. The only thing to do is to try one way — if it works, fine. If it does not, try the other. It is perhaps most natural to begin by looking for stories to pry the the premises off the conclusion. If you can find a consistent story to make the premises true and conclusion false, the argument is invalid. If you cannot find any such story, you may begin to suspect that the argument is valid. This suspicion does not itself amount to a demonstration of validity! But you might try to turn your suspicion into such a demonstration by attempting the validity method. Again, if one procedure works, the other better not!

E2.2. Use our validity procedure to show that each of the following is logically valid, and to decide (if you can) whether it is logically sound.

- a. If Bill is president, then Hillary is first lady

Hillary is not first lady

Bill is not president

- b. Only fools find love

Elvis was no fool

Elvis did not find love

- c. If there is a good and omnipotent god, then there is no evil

There is evil

There is no good and omnipotent god

- d. All Sparrows are birds

All birds fly

All sparrows fly

- e. All citizens can vote

Hannah is a citizen

Hannah can vote

E2.3. Use our procedures to say whether the following are logically valid or invalid, and sound or unsound. Hint: You may have to do some experimenting to decide whether the arguments are logically valid or invalid — and so to decide which procedure applies.

- a. If Bill is president, then Hillary is first lady

Bill is president

Hillary is first lady

- b. Most professors are insane

TR is a professor

TR is insane

- c. Some dogs have red hair

Some dogs have long hair

Some dogs have long red hair

- d. If you do not strike the match, then it will not light

The match lights

You strike the match

- e. Shaq is taller than Kobe

Kobe is at least as tall as TR

Kobe is taller than TR

3 Some Consequences

We now know what logical validity and soundness are and should be able to identify them in simple cases. Still, it is one thing to know what validity and soundness are, and another to know how we can use them. So in this section I turn to some consequences of the definitions.

3.1 Soundness and Truth

First, a consequence we want: The conclusion of every sound argument is true in the real world. Observe that this is *not* part of what we require to show that an argument is sound. *LS* requires just that an argument is valid and that its

premises are true. However, it is a consequence of these requirements that the conclusion is true as well. To see this, suppose we have a sound two-premise argument, and think about the nature of the true story. The premises and conclusion must fall into one of the following combinations of true and false in the real world:

1	2	3	4	5	6	7	8
T	T	T	F	T	F	F	F
$\frac{T}{T}$	$\frac{T}{F}$	$\frac{F}{T}$	$\frac{T}{T}$	$\frac{F}{F}$	$\frac{T}{F}$	$\frac{F}{T}$	$\frac{F}{F}$

If the argument is logically sound, it is logically valid; so no consistent story makes the premises true and the conclusion false. But the true story is a consistent story. So we can be sure that the true story does not result in combination (2). So far, the true story might fall into any of the other combinations. Thus the conclusion of a valid argument may or may not be true in the real world. But if an argument is sound, its premises are true in the real world. So, for a sound argument, we can be sure that the premises do not fall into any of the combinations (3) - (8). (1) is the only combination left: in the true story, the conclusion is true. And, in general, if an argument is sound, its conclusion is true in the real world: If there is no consistent story where the premises are true and the conclusion is false, and the premises are in fact true, then the conclusion must be true as well. Note again: we do not need that the conclusion is true in the real world in order to say that an argument is sound, and saying that the conclusion is true is no part of our procedure for validity or soundness! Rather, by discovering that an argument is logically valid and that its *premises* are true, we *establish* that it is sound; this gives us the result that its conclusion therefore is true. And that is just what we want.

3.2 Validity and Form

Some of the arguments we have seen so far are of the same general *form*. Thus both of the arguments on the left have the form on the right.

(G)	If Joe works hard, then he will get an 'A'	If Hannah is a citizen then she can vote	If \mathcal{P} then \mathcal{Q}
	$\frac{\text{Joe works hard}}{\text{Joe will get an 'A'}}$	$\frac{\text{Hannah is a citizen}}{\text{Hannah can vote}}$	$\frac{\mathcal{P}}{\mathcal{Q}}$

As it turns out, all arguments of this form are valid. In contrast, the following arguments with the indicated form are not.

(H)	If Joe works hard then he will get an ‘A’ <hr style="width: 100px; margin: 5px 0;"/> Joe will get an ‘A’ <hr style="width: 100px; margin: 5px 0;"/> Joe works hard	If Hannah can vote, then she is a citizen <hr style="width: 100px; margin: 5px 0;"/> Hannah is a citizen <hr style="width: 100px; margin: 5px 0;"/> Hannah can vote	If \mathcal{P} then \mathcal{Q} $\frac{\mathcal{Q}}{\mathcal{P}}$
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There are stories where, say, Joe cheats for the ‘A’, or Hannah is a citizen but not old enough to vote. In these cases, there is some other way to obtain condition \mathcal{Q} than by having \mathcal{P} — this is what the stories bring out. And, generally, it is often possible to characterize arguments by their forms, where a form is *valid* iff every instance of it is logically valid. Thus the first form listed above is valid, and the second not.

For now, it is worth noting that some presentations of critical reasoning (which you may or may not have encountered), take advantage of such patterns, listing typical ones that are valid, and typical ones that not (for example, Cederblom and Paulsen, *Critical Reasoning*). A student may then identify valid and invalid arguments insofar as they match the listed forms. This approach has the advantage of simplicity — and one may go quickly to applications of the logical notions to concrete cases. But the approach is limited to application of listed forms, and so to a very limited range, where our definition has application to arbitrary arguments. Further, a mere listing of valid forms does not explain their relation to truth, where the definition is directly connected. Formal logic also takes advantage of of certain very general structural features of arguments for an account of validity. But that approach attempts an account for arbitrary forms and so goes well beyond the mere lists of the more traditional approaches.³

3.3 Relevance

Another consequence seems less welcome. Consider the following argument.

³For an introduction to formal logic, see, for example, my *Symbolic Logic*. Some authors introduce a notion of *formal validity* (maybe in the place of logical validity as above) such that an argument is formally valid iff it has some valid form. As above, formal validity is parasitic on logical validity, together with a to-be-specified notion of form. But if an argument is formally valid, it is logically valid. So if one is in a position to identify formal validity, one identifies logical validity as well.

Snow is white

- (I) Snow is not white
All dogs can fly

It is natural to think that the premises are not connected to the conclusion in the right way — for the premises have nothing to do with the conclusion — and that this argument therefore should not be logically valid. But if it is not valid, by definition, there is a consistent story that makes the premises true and the conclusion false. And, in this case, there is no such story, for *no consistent story makes the premises true*. Thus, by definition, this argument is logically valid. The procedure applies in a straightforward way. Thus,

- a. In any story with premises true and conclusion false,
 - (1) Snow is white
 - (2) Snow is not white
 - (3) Some dogs cannot fly
- b. In any such story,

Given (1) and (2),

 - (4) Snow is and is not white
- c. So any story with premises true and conclusion false is inconsistent, and by definition the argument is logically valid.
- d. Since in the real world snow is white, not all the premises are true (the second premise is false). So, though the argument is logically valid, by definition it is not logically sound.

This seems bad! Intuitively, there is something wrong with the argument. But, on our official definition, it is logically valid. One might rest content with the observation that, even though the argument is logically valid, it is not logically sound. But this does not remove the general worry. For this argument,

- (J) There are fish in the sea
 $1 + 1 = 2$

has all the problems of the other and is logically *sound* as well. (Why?) One might, on the basis of examples of this sort, decide to reject the (classical)

account of validity with which we have been working. Some do just this.⁴ But, for now, let us see what can be said in defense of the classical approach. (And the classical approach is, no doubt, the approach you have seen or will see in any standard course on critical thinking or logic.)

As a first line of defense, one might observe that the conclusion of every sound argument is true and ask, “What more do you want?” We use arguments to demonstrate the truth of conclusions. And nothing we have said suggests that sound arguments do not have true conclusions: An argument whose premises are inconsistent, is sure to be unsound. And an argument whose conclusion cannot be false, is sure to have a true conclusion. So soundness may seem sufficient for our purposes. Even though we accept that there remains something about argument goodness that soundness leaves behind, we can insist that soundness is useful as an intellectual tool. Whenever it is the truth or falsity of a conclusion that matters, we can profitably employ the classical notions.

But one might go further, and dispute even the suggestion that there is something about argument goodness that soundness leaves behind. Consider the following two argument forms.

$$\begin{array}{ll} \text{(ds)} & \frac{\mathcal{P} \text{ or } \mathcal{Q}, \text{ not-}\mathcal{P}}{\mathcal{Q}} \qquad \qquad \text{(add)} \quad \frac{\mathcal{P}}{\mathcal{P} \text{ or } \mathcal{Q}} \end{array}$$

According to ds (*disjunctive syllogism*), if you are given that \mathcal{P} or \mathcal{Q} and that not- \mathcal{P} , you can conclude that \mathcal{Q} . If you have cake or ice cream, and you do not have cake, you have ice cream; if you are in California or New York, and you are not in California, you are in New York; and so forth. Thus ds seems hard to deny. And similarly for add (*addition*). Where ‘or’ means “one or the other or both,” when you are given that \mathcal{P} , you can be sure that \mathcal{P} or anything. Say you have cake, then you have cake or ice cream, cake or brussels sprouts, and so forth; if grass is green, then grass is green or pigs have wings, grass is green or dogs fly, and so forth.

Return now to our problematic argument. As we have seen, it is valid according to the classical definition LV. We get a similar result when we apply the ds and add principles.

⁴Especially the so-called “relevance” logicians. For an introduction, see Graham Priest, *Non-Classical Logics*. But his text presumes mastery of material corresponding to ?? and ?? (or at least ?? with ??) of this one. So the non-classical approaches develop or build on the classical one developed here.

- | | |
|--------------------------------------|---------------------|
| 1. Snow is white | premise |
| 2. Snow is not white | premise |
| 3. Snow is white or all dogs can fly | from 1 and add |
| 4. All dogs can fly | from 2 and 3 and ds |

If snow is white, then snow is white or anything. So snow is white or dogs fly. So we use line 1 with *add* to get line 3. But if snow is white or dogs fly, and snow is not white, then dogs fly. So we use lines 2 and 3 with *ds* to reach the final result. So our principles *ds* and *add* go hand-in-hand with the classical definition of validity. The argument is valid on the classical account; and with these principles, we can move from the premises to the conclusion. If we want to reject the validity of this argument, we will have to reject not only the classical notion of validity, but also one of our principles *ds* or *add*. And it is not obvious that one of the principles should go. If we decide to retain both *ds* and *add* then, seemingly, the classical definition of validity should stay as well. If we have intuitions according to which *ds* and *add* should stay, and also that the definition of validity should go, we have conflicting intuitions. Thus our intuitions might, at least, be sensibly resolved in the classical direction.

These issues are complex, and a subject for further discussion. For now, it is enough for us to treat the classical approach as a useful tool: It is useful in contexts where what we care about is whether conclusions are true. And alternate approaches to validity typically develop or modify the classical approach. So it is natural to begin where we are, with the classical account. At any rate, this discussion constitutes a sort of acid test: If you understand the validity of the “snow is white” and “fish in the sea” arguments (I) and (J), you are doing well — you understand *how* the definition of validity works, with its results that may or may not now seem controversial. If you do not see what is going on in those cases, then you have not yet understood how the definitions work and should return to section 2 with these cases in mind.

E3.1. Use our procedures to say whether the following are logically valid or invalid, and sound or unsound. Hint: You may have to do some experimenting to decide whether the arguments are logically valid or invalid — and so to decide which procedure applies.

- a. Bob is over six feet tall
Bob is under six feet tall
 Bob is disfigured

- b. Marilyn is not over six feet tall
 Marilyn is not under six feet tall
 Marilyn is beautiful
- c. There are fish in the sea
 There are birds in the sky
 There are bats in the belfry
 Two dogs are more than one
- d. The earth is (approximately) round
 There is no round square
- e. All dogs can fly
 Fido is a dog
 Fido cannot fly
 I am blessed

E3.2. Respond to each of the following.

- a. Create another argument of the same form as the first set of examples (G) from section 3.2, and then use our regular procedures to decide whether it is logically valid and sound. Is the result what you expect? Explain.
- b. Create another argument of the same form as the second set of examples (H) from section 3.2, and then use our regular procedures to decide whether it is logically valid and sound. Is the result what you expect? Explain.

E3.3. Which of the following are true, and which are false? In each case, explain your answers, with reference to the relevant definitions. The first is worked as an example.

- a. A logically valid argument is always logically sound.
 False. An argument is sound iff it is logically valid and all of its premises are true in the real world. Thus an argument might be valid but fail to be sound if one or more of its premises is false in the real world.

- b. A logically sound argument is always logically valid.
- c. If the conclusion of an argument is true in the real world, then the argument must be logically valid.
- d. If the premises and conclusion of an argument are true in the real world, then the argument must be logically sound.
- e. If a premise of an argument is false in the real world, then the argument cannot be logically valid.
- f. If an argument is logically valid, then its conclusion is true in the real world.
- g. If an argument is logically sound, then its conclusion is true in the real world.
- h. If an argument has contradictory premises (its premises are true in no consistent story), then it cannot be logically valid.
- i. If the conclusion of an argument cannot be false (is false in no consistent story), then the argument is logically valid.
- j. The premises of every logically valid argument are relevant to its conclusion.

E3.4. For each of the following concepts, explain in an essay of about two pages, so that Hannah could understand. In your essay, you should (i) identify the objects to which the concept applies, (ii) give and explain the definition, and give and explicate examples, preferably of your own construction, (iii) where the concept applies, and (iv) where it does not. Your essay should exhibit an understanding of methods from the text.

- a. Logical validity
- b. Logical soundness

E3.5. Do you think we should accept the classical account of validity? In an essay of about two pages, explain your position, with special reference to difficulties raised in section 3.3.

4 Answers to Selected Exercises

E1.1. Say whether each of the following stories is internally consistent or inconsistent. In either case, explain why.

- a. Smoking cigarettes greatly increases the risk of lung cancer, although most people who smoke cigarettes do not get lung cancer.

Consistent. Even though the risk of cancer goes up with smoking, it may be that most people who smoke do not have cancer. Perhaps 49% of people who smoke get cancer, and 1% of people who do not smoke get cancer. Then smoking greatly increases the risk, even though most people who smoke do not get it.

- c. Abortion is always morally wrong, though abortion is morally right in order to save a woman's life.

Inconsistent. Suppose (whether you believe it or not) that abortion is *always* morally wrong. Then it is wrong to save a woman's life. So the story requires that it is and is not wrong to save a woman's life.

- e. No rabbits are nearsighted, though some rabbits wear glasses.

Consistent. One reason for wearing glasses is to correct nearsightedness. But glasses may be worn for other reasons. It might be that rabbits who wear glasses are farsighted, or have astigmatism, or think that glasses are stylish. Or maybe they wear sunglasses just to look cool.

- g. Bill Clinton was never president of the United States, although Hillary is president right now.

Consistent. Do not get confused by the facts! In a story it may be that Bill was never president and his wife was. Thus this story does not contradict itself and is consistent.

- i. The death star is a weapon more powerful than that in any galaxy, though there is, in a galaxy far far away, a weapon more powerful than it.

Inconsistent. If the death star is more powerful than any weapon in any galaxy, then according to this story it is and is not more powerful than the weapon in the galaxy far far away.

- E1.2. For each of the following sentences, (i) say whether it is true or false in the real world and then (ii) say if you can whether it is true or false according to the accompanying story. In each case, explain your answers.
- a. Sentence: Aaron Burr was never a president of the United States.
Story: Aaron Burr was the first president of the United States, however he turned traitor and was impeached and then executed.
(i) It is *true* in the real world that Aaron Burr was never a president of the United States. (ii) But the story makes the sentence *false*, since it says that he was the first president.
- c. Sentence: After overrunning Phoenix, the buffalo did in Newark, New Jersey.
Story: A thundering herd of buffalo overran Phoenix Arizona in early 2006. The city no longer exists.
(i) It is *false* in the real world that any herd of buffalo overran Newark anytime after 2006. (ii) And, though the story says something about Phoenix, the story does not contain enough information to say whether the sentence regarding Newark is true or false.
- e. Sentence: Jack Nicholson has swum the Atlantic.
Story: No human being has swum the Atlantic. Jack Nicholson and Bill Clinton and you are all human beings, and at least one of you swam all the way across!
(i) It is *false* in the real world that Jack Nicholson has swum the Atlantic. (ii) This story is inconsistent! It requires that some human both has and has not swum the Atlantic. Thus we refuse to say that it makes the sentence true or false.
- g. Sentence: Your instructor is not a human being.
Story: Aliens have never made it to this country. However, without a doubt, your instructor is from another planet.
(i) Presumably, the claim that your instructor is not a human being is *false* in the real world (assuming that you are not working by independent, or computer-aided study). (ii) But this story is inconsistent! It says both that aliens have and have not made it to another planet. Thus we refuse to say that it makes any sentence true or false.

- i. Sentence: The Yugo is the most expensive car in the world.

Story: Jaguar and Rolls Royce are expensive cars. But the Yugo is more expensive than either of them.

(i) The Yugo is a famously cheap automobile. So the sentence is *false* in the real world. (ii) According to the story, the Yugo is more expensive than some expensive cars. But this is not enough information to say whether it is the most expensive car in the world. So there is not enough information to say whether the sentence is true or false.

- E2.1. Use our invalidity test to show that each of the following arguments is not logically valid, and so not logically sound.

*For each of these problems, different stories might do the job.

- a. If Joe works hard, then he will get an 'A'

Joe will get an 'A'

Joe works hard

- a. In any story with premises true and conclusion false,

1. If Joe works hard, then he will get an 'A'
2. Joe will get an 'A'
3. Joe does not work hard

- b. Story: Joe is very smart, and if he works hard, then he will get an 'A'. Joe will get an 'A'; however, Joe cheats and gets the 'A' without working hard.

- c. This is a consistent story that makes the premises true and the conclusion false; thus, by definition, the argument is not logically valid.

- d. Since the argument is not logically valid, by definition, it is not logically sound.

- E2.2. Use our validity procedure to show that each of the following is logically valid, and to decide (if you can) whether it is logically sound.

*For each of these problems, particular reasonings might take different forms.

- a. If Bill is president, then Hillary is first lady

Hillary is not first lady

Bill is not president

- a. In any story with premises true and conclusion false,

(1) If Bill is president, then Hillary is first lady

(2) Hillary is not first lady

(3) Bill is president

- b. In any such story,

Given (1) and (3),

(4) Hillary is first lady

Given (2) and (4),

(5) Hillary is and is not first lady

- c. So any story with premises true and conclusion false is inconsistent and, by definition, the argument is logically valid.

- d. In the real world Hillary is not first lady and Bill and Hillary are married so it is true that if Bill is president, then Hillary is first lady; so all the premises are true and by definition the argument is logically sound.

E2.3. Use our procedures to say whether the following are logically valid or invalid, and sound or unsound. Hint: You may have to do some experimenting to decide whether the arguments are logically valid or invalid — and so to decide which procedure applies.

- c. Some dogs have red hair

Some dogs have long hair

Somme dogs have long red hair

- a. In any story with the premise true and conclusion false,

1. Some dogs have red hair

2. Some dogs have long hair

3. No dogs have long red hair

- b. Story: There are dogs with red hair, and there are dogs with long hair. However, due to a genetic defect, no dogs have long red hair.
- c. This is a consistent story that makes the premise true and the conclusion false; thus, by definition, the argument is not logically valid.
- d. Since the argument is not logically valid, by definition, it is not logically sound.

E3.1. Use our procedures to say whether the following are logically valid or invalid, and sound or unsound.

- d. The earth is (approximately) round

There is no round square

- a. In any story with the premise true and conclusion false,
 - (1) The earth is (approximately) round
 - (2) There is a round square
- b. In any such story, given (2),
 - (3) Something is round and not round
- c. So any story with premise true and conclusion false is inconsistent and, by definition, the argument is logically valid.
- d. In the real world the earth is (approximately) round, so the premise is true and by definition the argument is logically sound.

E3.3. Which of the following are true, and which are false? In each case, explain your answers, with reference to the relevant definitions.

- c. If the conclusion of an argument is true in the real world, then the argument must be logically valid.

False. An argument is logically valid iff there is no consistent story that makes the premises true and the conclusion false. Though the conclusion is true in the real world (and so in the true story), there may be some other story that makes the premises true and the conclusion false. If this is so, then the argument is not logically valid.

- e. If a premise of an argument is false in the real world, then the argument cannot be logically valid.

False. An argument is logically valid iff there is no consistent story that makes the premises true and the conclusion false. For logical validity, there is no requirement that every story have true premises — only that ones that do, also have true conclusions. So an argument might be logically valid, and have premises that are false in many stories, including the true story.

- g. If an argument is logically sound, then its conclusion is true in the real world.

True. An argument is logically valid iff there is no consistent story that makes the premises true and the conclusion false. An argument is logically sound iff it is logically valid and its premises are true in the real world. Since the premises are true in the real world, they hold in the true story; since the argument is valid, this story cannot be one where the conclusion is false. So the conclusion of a sound argument is true in the real world.

- i. If the conclusion of an argument cannot be false (is false in no consistent story), then the argument is logically valid.

True. If there is no consistent story where the conclusion is false, then there is no consistent story where the premises are true and the conclusion is false; but an argument is logically valid iff there is no consistent story where the premises are true and the conclusion is false. So the argument is logically valid.

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